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# An OLG Model of Common Ownership: Effects on Consumption and Investments\*

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## Abstract

We analyze how an increase in the degree of common ownership of firms in the same market affects consumption and investment. Such an increase is shown to reduce real investment and therefore intertemporal consumption. Overall, institutional investors' common ownership of firms competing in the same market serves as a device for weakening market competition. The resulting increase in the price of acquiring shares with institutional investors then crowds out savings directed to real investments.

**Keywords:** Common ownership, institutional investors, real versus financial investments, market power, savings and investments, investment crowding-out, overlapping generations.

**JEL Classification Number:** G11, G23, L13, L41

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## 1. Introduction

Institutional investors (such as mutual and pension funds) hold an increasingly high share of US publicly traded firms. Azar, Schmalz, and Tecu (2018) estimate it to be in the 70–80 percent range, and Backus, Conlon, and Sinkinson (2019a) point out that in 2018 one of the four largest asset managers (Blackrock, Vanguard, State Street and Fidelity) was the largest shareholder for 88 percent of firms on the S&P 500 Index. As pointed out in several recent studies, frequently, such institutional investors have considerable ownership stakes in several firms competing in the same industry. He and Huang (2017) present evidence showing an increase, from below 10 percent in 1980 to about 60 percent in 2014, in the fraction of US public firms with common institutional ownership, such that the institutional owner simultaneously holds at least 5 percent of the common equity of other firms in the same industry. Similarly, according to Azar (2016), the share of S&P 500 firms having overlapping owners with at least 3 percent ownership stakes in firms operating in the same industry has increased from 25 percent to 90 percent in the decade between years 2000 and 2010.<sup>1</sup> Seldeslachts, Newham, and Banal-Estanol (2017) document an increasing trend of common ownership in Germany. However, in Germany, the pattern of common ownership by institutional owners is not a general economy-wide phenomenon and is restricted to specific industries, such as the chemical industry.

Some recent studies examine the hypothesis that a higher degree of common ownership relaxes competition. Azar, Schmalz, and Tecu (2018) and Azar, Raina, and Schmalz (2016) present empirical evidence from the US airline and banking industries, suggesting that prices in these industries have risen in response to increased common ownership. Theoretical models exploring the effects of common ownership or overlapping ownership on various dimensions of market performance include O’Brien and Salop (2000), López and Vives (Forthcoming), Newham, Seldeslachts, and Banal-Estañol (2018), and Vives (2019). Shy and Stenbacka (2019b) analyze the tradeoff between relaxed competition and enhanced diversification introduced by a higher degree

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<sup>1</sup>According to Zingales (2012) (p.233), in the 1920s individuals owned 90-percent of publicly traded equity. By 2007, that figure had dropped to less than 30 percent, where most of this share is represented by management and insiders who collectively own 24 percent of the equity in a typical company. During the same period, the percentage of US equity owned by institutions has risen from less than 10 percent to more than 60 percent. Therefore, almost all daily trading in US stocks is conducted by institutional investors.

of common ownership within an intraindustry framework with risk aversion. Motivated by these research approaches, economists as well as legal scholars have formulated policy proposals. Elhauge (2016) and Posner, Scott Morton, and Weyl (2017) propose rules restricting the possibilities for institutional owners to hold ownership stakes in several firms operating in the same industry.

This study analyzes the implications of common ownership from a perspective very different from the earlier ones by addressing the following research questions: (i) What are the effects of increased common ownership on savings, intertemporal consumption, and welfare under circumstances where individuals with a finite lifetime allocate their savings between institutional investors maintaining ownership in several firms competing in the same market? (ii) How does such common ownership affect the allocation of resources between financial investments to acquire ownership of institutional investors and real investments that enhance resources available to future generations? Analyzing these questions is important because individuals channel a significant proportion of their long-term savings into pension funds. These funds constitute a significant portion of the institutional investors described above.

We design an overlapping generations (OLG) model, where in each period the young consumers determine the allocation of their savings between consumption, acquisition of shares with institutional investors, and real investment financed via interest-bearing bonds. The real investment is assumed to promote the endowment of resources available to the subsequent generation. At old age, consumers collect dividends paid by institutional investors, sell their ownership shares to the young of a new generation, and collect their principal and interest on bonds, all to support old-age consumption.

We demonstrate that real investments decrease in response to a higher degree of common ownership that the institutional investors hold in firms producing in the same industry. Also, in a steady state equilibrium, the consumption of the young as well as old generation of consumers decreases with the degree of common ownership as long as the return on real investments exceeds that of holding shares in the institutional investors.

Common insights gained from static oligopoly models typically emphasize the uncontroversial view that weak competition tends to hurt consumers. Such a perspective focuses on com-

petition between firms operating in the same relevant market and on consumption opportunities restricted to the relevant market in question. The present study adds another dimension by analyzing consumers in the role of investors seeking to optimize the net present value of lifetime consumption. More precisely, it focuses on consumers in their role as shareholders of institutional investors, say pension funds, under circumstances where these individuals have consumption opportunities outside the industry under consideration. We find institutional investors' common ownership of firms competing in the same market to be a device for weakening price competition, which consequently diverts savings from real investments to the acquisition of financial ownership. The increased value of financial assets crowds out real investment, thereby reducing lifetime consumption and hence welfare. In other words, common ownership of firms competing in the same market generates a distortion in the intertemporal resource allocation of consumers, and this distortion adds to the traditional welfare loss familiar from static models of cross-ownership between firms operating in oligopolistic markets.

It is important to emphasize the distinction between a model of institutional investors' common ownership of product market firms, and a model of cross ownership where each product market firm owns a (minority) equity share in a rival firm. The existing theoretical literature in industrial organization suggests that cross ownership tends to reduce competition no matter whether it focuses on the unilateral effects (see, Reynolds and Snapp (1986) or Farrell and Shapiro (1990)) or the coordinated effects (see, Gilo, Moshe, and Spiegel (2006)). These theoretical predictions are also supported by empirical evidence, see Nain and Wang (2016). However, by design, the models evaluating the effects of cross ownership cannot evaluate how ownership links among firms affect the allocation of resources by consumers with finite lifetime. Our model is designed precisely to evaluate effects of such ownership links on consumption, real investments, and acquisition of ownership of institutional investors with shares in competing same-industry firms.

This study is organized as follows. Section 2 designs a static duopoly model in order to measure how the share value of institutional investors varies with the degree of their common ownership in firms competing in the same product market. Section 3 constructs an overlapping generations model of consumers who allocate their savings among young-age consumption, buy-

ing shares of institutional investors, and interest-bearing bond-financed real investments. Section 4 derives the equilibrium share value of institutional investors, allocation of savings to bond-financed real investment, and lifetime consumption. Section 5 presents the main results by characterizing the effects of varying the degree of common ownership on consumption and real investment. Section 6 extends the model to uncertain returns. Section 7 presents concluding comments. Appendices provide algebraic derivations for some results.

## 2. Institutional investors, common ownership, and competition

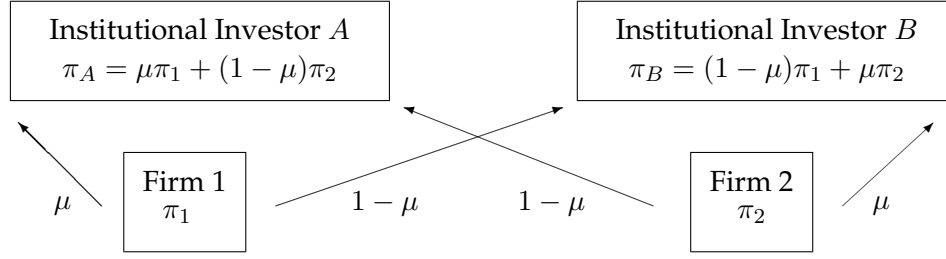
We introduce institutional investors into a modified duopoly model with two firms competing based on production decisions in the product market. The two firms are co-owned by institutional investors that hold shares in both producing firms. This section investigates how the ownership value of institutional investors is influenced by the degree of their common ownership of the producing firms. The results obtained in this section serve as a benchmark of institutional investors' per-period performance in an infinite-horizon overlapping generations (OLG) model to be introduced in Section 3. Algebraic derivations for this section are relegated to Appendix A.

Consider two producing firms, firm 1 and firm 2, engaged in Cournot quantity competition. The variables  $q_1$  and  $q_2$  denote the quantities of a homogeneous good (or service) produced and sold by firm 1 and 2, respectively. The aggregate inverse demand function is  $p = \alpha - \beta(q_1 + q_2)$ , where  $\alpha, \beta > 0$ . Let  $\pi_1$  and  $\pi_2$  denote the profits earned by firms 1 and 2, respectively. Then, assuming zero marginal costs, the producing firms' profits as functions of quantity produced are given by

$$\pi_1(q_1, q_2) = pq_1 = [\alpha - \beta(q_1 + q_2)] q_1 \quad \text{and} \quad \pi_2(q_1, q_2) = pq_2 = [\alpha - \beta(q_1 + q_2)] q_2. \quad (1)$$

Firms 1 and 2 (the producers) are co-owned by two institutional investors labeled  $A$  and  $B$ , as illustrated in Figure 1 and formalized in Assumption 1. Investor  $A$  owns a share  $\mu$  in firm 1, and  $(1 - \mu)$  in firm 2. Similarly, investor  $B$  owns a share  $\mu$  in firm 2, and  $(1 - \mu)$  in firm 1.

**ASSUMPTION 1.** *Institutional investor  $A$  owns a majority share  $\mu$  in firm 1 and institutional investor  $B$  owns a majority share  $\mu$  in firm 2, where  $\mu \in (\frac{1}{2}, 1]$ .*



**Figure 1:** The shares of ownership in firms 1 and 2 by institutional investors  $A$  and  $B$ .

To simplify, we assume that institutional investors  $A$  and  $B$  are the sole owners of firms 1 and 2. Therefore, Assumption 1 implies that institutional investor  $A$  owns a minority share ( $1 - \mu < 50\%$ ) in firm 2, whereas institutional investor  $B$  owns a minority share ( $1 - \mu < 50\%$ ) in firm 1. In view of Figure 1 and Assumption 1, the profits earned by the institutional investors, as functions of quantity produced by firms 1 and 2, are  $\pi_A = \mu\pi_1 + (1 - \mu)\pi_2$  and  $\pi_B = (1 - \mu)\pi_1 + \mu\pi_2$ , where  $\pi_1$  and  $\pi_2$  are defined in (1).

The literature does not provide a consistent method or a consensus regarding the modeling of how ownership actually translates into control of firms' decisions. OECD (2017) and Backus, Conlon, and Sinkinson (2019b) present extensive discussions of how firms in the presence of common ownership can apply profit weights as a mechanism to internalize strategic externalities in the product market. Because institutional investor  $A$  is the majority shareholder in firm 1,  $A$  can determine firm 1's output level either through the exercise of direct influence or through the control of underlying managerial incentives. This means that investor  $A$  controls the production of firm 1, while taking into consideration the profit derived from its minority ownership share in firm 2. Similarly, institutional investor  $B$  determines the output level produced by firm 2 taking into account its minority ownership share in firm 1.<sup>2</sup> Therefore, institutional investors  $A$  and  $B$

<sup>2</sup>An alternative modeling method would be to assume that a firm's production decision is made in order to maximize the total portfolio value of its investors, weighted by the proportion of ownership held by these investors. Such an approach has been applied by O'Brien and Salop (2000). The associated distinction between profit maximization and shareholder value maximization and its role for the analysis of strategic competition is discussed in Antón et al. (2018).

solve (separately) the following profit-maximization problems:

$$\begin{aligned}\max_{q_1} \pi_A(q_1, q_2) &= \mu\pi_1(q_1, q_2) + (1 - \mu)\pi_2(q_1, q_2), \\ \max_{q_2} \pi_B(q_1, q_2) &= (1 - \mu)\pi_1(q_1, q_2) + \mu\pi_2(q_1, q_2).\end{aligned}\tag{2}$$

Appendix A derives the following Cournot-Nash equilibrium production levels, the corresponding price, and profits.

$$q_1 = q_2 = \frac{\alpha\mu}{\beta(2\mu + 1)}, \quad p = \frac{\alpha}{2\mu + 1}, \quad \text{and} \quad \pi_A = \pi_B = \pi_1 = \pi_2 = \frac{\gamma\mu}{(2\mu + 1)^2},\tag{3}$$

where  $\gamma = \alpha^2/\beta$ . The equilibrium properties are summarized as follows:

- Result 1.** (a) *Market price and profit earned by each institutional investor decline with increased majority shares of both investors. Formally,  $\partial p/\partial\mu < 0$ ,  $\partial\pi_A/\partial\mu < 0$ , and  $\partial\pi_B/\partial\mu < 0$ .*
- (b) *Market price and profits are maximized when both institutional investors hold equal shares in each producing firm ( $\mu = \frac{1}{2}$ ), and minimized when each institutional investor maintains 100-percent ownership in one firm ( $\mu = 1$ ).*

We can interpret Result 1(b) to imply that a fully diversified ownership structure maximizes profits, whereas concentration of ownership on behalf of each institutional owner to one firm minimizes profits. The intuitive reason is that the diversified ownership structure aligns the interests of the two competing institutional owners, thereby supporting a maximal price at a minimal industry production level. Thus, diversified ownership leads to weak competition at the expense of consumers. For the same reason diversified ownership benefits the institutional investors as owners of the competing producers.

### 3. An OLG model of common ownership by institutional investors

In each period  $t = 0, 1, 2, \dots$ , the economy consists of two representative consumers, a young one and an old one, as well as two institutional investors that own shares in the two competing producing firms analyzed in Section 2. The institutional investors are owned exclusively by the consumers. At young age, consumers can acquire ownership shares in the institutional investors.



This means that the institutional investors also serve as pension funds offering an investment opportunity whereby young individuals can save for old age consumption.

Young consumers also have the option of allocating their savings to bonds. The bonds are issued for the sake of financing real investments, which induce growth in the endowment available for consumption by the next generation of young consumers. We consider such bonds to represent real investment in the economy. Throughout our study we will make use of the following terminology:

DEFINITION 1. *We say that investment*

- (a) *is **financial** if it focuses on transfer of ownership of profit-making firms, and*
- (b) *is **real** if it enhances the endowment available to the young consumers of the next generation.*

In view of Definition 1, investments to acquire ownership in the institutional investors will be referred to as *financial*, as these institutional investors themselves focus on acquiring ownership of the existing producing firms analyzed in Section 2. It should be emphasized that the purpose of our analysis is to characterize the effects of changing the degree of common ownership of producing firms by institutional investors ( $\mu$ ) on consumption and investments. In order to conduct our analysis in a tractable manner, we abstract from equity investments whereby firms could issue new capital to fund real investment programs. Essentially, in the context of our model, this amounts to a view according to which the producing firms operate in a stationary market environment with a fixed technology that does not depreciate.

A *real* investment is financed by bonds issued by the public sector or by private organizations. The central feature is that these bonds finance investments made for the purpose of enhancing the endowment of the young consumers belonging to the next generation. Investments targeting education, health services, and generation-specific infrastructure provide good examples.

The following subsections characterize the young consumer's allocation of resources between financial and real investments in greater detail.

### 3.1 The investment allocation of young consumers

The period  $t$  young consumer is endowed with  $\omega_t$  units of a real resource that also serves as the unit of account and exchange. The endowment of a young consumer can be used for three purposes: consumption when young, acquisition of shares with institutional investors (financial investment), and investment in bonds that finance real investments. Initially, the shares of institutional investors are held (owned) by the old consumer. Formally, the resource constraint faced by a period  $t$  representative young consumer is given by

$$c_t^y + x_t v_t + b_t = \omega_t, \quad t = 0, 1, \dots, \quad (4)$$

where  $c_t^y$  is the period  $t$  consumption by the young consumer,  $v_t$  is the period  $t$  aggregate value of the institutional investors' shares, and  $b_t$  is a real investment via bond financing that matures and pays interest  $r \geq 0$  in period  $t + 1$ . The choice variable  $x_t \in [0, 1]$  is the fraction of the share value  $v_t$  purchased by period  $t$  young consumer from the period  $t$  old consumer.

The consumption good in each period is assumed to be a composite good of a representative bundle of goods, denominated in the unit of account which is normalized to equal 1. We make the following assumption.

*ASSUMPTION 2. The good/service produced by firms 1 and 2 (analyzed in Section 2) constitutes a negligible fraction of consumers' composite consumption bundles,  $c_t^y$  (young at  $t$ ) and  $c_t^o$  (old at  $t$ ).*

Assumption 2 facilitates the separation of consumers' acquisition of ownership of the institutional investors from the price effects in the product market where the producing firms are owned by these institutional investors. Thus, the value derived from the acquisition of shares in institutional investors is independent of the product/service sold by the firms owned by these institutional investors.

### 3.2 Old consumers, trade between generations, and lifetime preferences

Old-age consumers trade all their assets to facilitate consumption prior to their exit from the market. More precisely, the representative old in period  $t + 1$  collects the dividends  $x_t d_{t+1}$  paid by institutional investors, sells all the shares owned in the institutional investors (valued  $x_t v_{t+1}$ ) to

the young in period  $t + 1$ , and collects the return on maturing bonds purchased at young age  $b_t(1 + r)$ . Therefore, generation  $t$ 's consumption when old in period  $t + 1$  is

$$c_{t+1}^o = x_t(d_{t+1} + v_{t+1}) + b_t(1 + r), \quad t = 0, 1, \dots \quad (5)$$

The representative young consumer of generation  $t$  ( $t = 0, 1, 2, \dots$ ) maximizes a two-period discounted utility from consumption given by

$$U^t = U(c_t^y, c_{t+1}^o) = u(c_t^y) + \delta u(c_{t+1}^o), \quad \text{where } u(c) = \begin{cases} \frac{1}{1-\theta} c^{1-\theta} & \text{if } \theta > 0 \text{ and } \theta \neq 1 \\ \ln(c) & \text{if } \theta = 1. \end{cases} \quad (6)$$

The parameter  $\delta \in (0, 1)$  is the time discount factor and  $u(c)$  is a constant relative risk aversion (CRRA) utility function.<sup>3</sup> In order to avoid duplication in presenting our results we focus our formal analysis on the case with  $\theta \neq 1$ . However, the analysis of the logarithmic utility function ( $\theta = 1$ ) is completely analogous with identical qualitative results, and an earlier version of this study focused on this special case.

This maximization assumes that the young consumer in period  $t$  has rational expectations in the sense of being able to anticipate the correct future values of dividends  $d_{t+1}$  and the share value  $v_{t+1}$ . These values are taken as given in the consumption resource constraints specified in (4) and (5). The initial representative old agent at  $t = 0$  (generation  $t = -1$ ) maximizes the second part of (6), which is equivalent to selling all resources to maximize consumption  $c_0^o$  according to (5).

### 3.3 Institutional investors

Consider the two institutional investors  $A$  and  $B$  analyzed in Section 2 where institutional investor  $A$  owns a majority share  $\mu$  in producer 1 and institutional investor  $B$  owns a majority share  $\mu$  in producer 2, for  $\mu \in (\frac{1}{2}, 1]$ . In order to quantify the consumption level of an old consumer, we make the following assumption about the dividend distributions.

**ASSUMPTION 3.** *In each period  $t$ , institutional investors distribute all their period  $t$  profits as dividends to shareholders. Formally,  $d_t = \Pi = \pi_A + \pi_B$ , for  $t = 0, 1, 2, \dots$ , where  $\pi_A$  and  $\pi_B$  are given in (3).*

Result 1(a) shows that the total per-period dividend distributions decrease when the institutional

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<sup>3</sup>The index of relative risk aversion for this utility function is  $-cu''(c)/u'(c) = \theta$ .

investors concentrate their assets to achieve a higher degree of majority ownership in the producing firms. This means that the dividends increase with the degree of common ownership. Dividends are maximized with diversified ownership such that institutional investors maintain equal (50 percent) shares in each producing firm.

The period  $\tau$  aggregate share value of the institutional investors is the present value of the discounted sum of all future profits they derive from the producers that they own starting from  $\tau + 1$ . Formally,

$$v_\tau = \sum_{t=\tau+1}^{\infty} \left( \frac{1}{1+r} \right)^{t-\tau} d_t = \frac{1}{1+r} (d_{\tau+1} + v_{\tau+1}). \quad (7)$$

The expression on the right in (7) is a recursive representation of institutional investors' aggregate share value. This value is expressed as the discounted sum of next period dividends plus the sale value of institutional shares, which the old in  $\tau + 1$  sell to the young in  $\tau + 1$ .

Substitution of (3) into expression (7) combined with Assumption 3 imply that the per-period dividend distributed by both institutional investors as well as their aggregate share value in each period are given by

$$d_t = \frac{2\gamma\mu}{(2\mu+1)^2} \quad \text{and} \quad v_t = \frac{2\gamma\mu}{r(2\mu+1)^2}, \quad (8)$$

respectively.

### 3.4 Bond-financed real investment

Real investment is financed by the public sector or a private organization that issue bonds to finance enhancements for the endowment received by next generation's young consumer. Formally, by issuing  $b_t$  worth of bonds to the young consumer in period  $t$ , the investing agency can boost the period  $t + 1$  endowment of generation  $t + 1$  so that

$$\omega_{t+1} = \omega + b_t(\rho - r), \quad \text{for } \rho \geq r. \quad (9)$$

This means that the net investment return on bond-financed real investment is  $\rho - r$  with  $\rho$  as the gross return and  $r$  as the interest rate paid to bond holders. Thus, with  $b_t$  worth of bonds issued in period  $t$ ,  $b_t(\rho - r)$  is the increase in the endowment of the period  $t + 1$  young representative consumer. The following assumption ensures that young consumers have sufficient re-

sources to purchase these bonds. It also guarantees that each generation allocates some resources to consumption, acquisition of ownership of institutional investors, and real investments. In other words, it rules out corner solutions.

ASSUMPTION 4. (a) *The endowment is sufficiently large. Formally for all  $\mu \in (\frac{1}{2}, 1]$ ,*

$$\omega > \frac{2\gamma\mu\delta^{-\frac{1}{\theta}} \left[ (1+r)^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}} \right]}{r(2\mu+1)^2}.$$

(b) *The return on bond-financed investment is bounded relative to the interest paid to bond holders.*

*Formally,  $r \leq \rho < 1+r$ .*

It is important to point out that the model assumes a process with non-cumulative endowment enhancement. Such a focus resembles real generation-specific investments in services such as education, health, and access to transportation as opposed to construction of durable infrastructure such as sewer and bridges that may last longer than one generation. This allows us to avoid the analytical complications resulting from continually growing endowment without losing the ability to investigate the effects of common ownership on financial investments, real investments, and consumption.

## 4. Equilibrium choice of financial and real investments

This section computes the equilibrium consumption and real investment paths  $\{c_t^y\}_0^\infty$ ,  $\{c_{t+1}^o\}_0^\infty$ , and  $\{b_t\}_0^\infty$ . Then, in Section 5 we characterize the effects of the degree of common ownership by institutional investors ( $\mu$ ) on consumption, real investment, and welfare.

### 4.1 Financial and real investments: Rates of return

In light of the resource constraint (4), the period  $t$  young consumer faces a choice of how to allocate savings between financial and real investments. Formally, the young can acquire a fraction  $x_t \in [0, 1]$  of the shares in institutional investors  $v_t$ , and also allocate resources  $b_t$  to public bonds bearing interest  $r$  in  $t+1$ . This optimization problem could potentially generate a corner solution where the young consumer allocates all savings to bonds and none to the acquisition of shares in institutional investors, or the other way around.

To formally address this issue, following Kehoe (1989) and McCandless and Wallace (1991), multiplying (4) by  $1 + r$  and adding (5) yields the future value of a generation's lifetime consumption

$$(1 + r)c_t^y + c_{t+1}^o = \omega_t(1 + r) + x_t \overbrace{[d_{t+1} + v_{t+1} - v_t(1 + r)]}^{\text{Excess return} = e_t}. \quad (10)$$

The term in the brackets measures excess return from buying all the shares in institutional investors valued at  $v_t$  over allocating the same amount to interest-bearing bonds. That is, if the young consumer acquires all shares in institutional investors  $v_t$ , the return when old consists of the sum of dividends  $d_{t+1}$  and the proceeds from selling the shares valued at  $v_{t+1}$  to the young of generation  $t + 1$ . If, instead, the young buys  $v_t$  worth of bonds, the return would be  $v_t(1 + r)$ . Therefore, if  $e_t > 0$ , savings via institutional investors strictly dominates bond purchase. In contrast, if  $e_t < 0$ , the return on bonds dominates that of institutional investors, in which case shares of institutional investors would not be traded between generations. Hence, we look for an equilibrium where both forms of savings coexist so that  $e_t = 0$  which is exactly the relationship given in (7). Under this condition, because both forms of saving yield the same return, we can set  $x_t = 1$  and look for an equilibrium where the entire value of institutional investments is traded between generations.

## 4.2 Young agents' optimization problem

Substituting  $x_t = 1$  into the resource constraints (4) and (5) facing the young and old, and into the two-period utility function (6), and then substituting (8), with a given endowment  $\omega_t$ , a representative young consumer in period  $t$  determines the savings allocated to interest-bearing bonds  $b_t$  to solve

$$\begin{aligned} \max_{b_t} U^t &= u(\overbrace{\omega_t - v_t - b_t}^{c_t^y}) + \delta u(\overbrace{d_{t+1} + v_{t+1} + b_t(1 + r)}^{c_{t+1}^o}) \\ &= \frac{1}{1 - \theta} \left[ \omega_t - \frac{2\gamma\mu}{r(2\mu + 1)^2} - b_t \right]^{1-\theta} + \frac{\delta}{1 - \theta} \left[ \frac{2\gamma\mu}{(2\mu + 1)^2} + \frac{2\gamma\mu}{r(2\mu + 1)^2} + b_t(1 + r) \right]^{1-\theta}. \end{aligned} \quad (11)$$

Appendix B derives the steady state solution for (11), yielding real investment via bond financing by the period  $t$  young consumer and, using (9), the resulting equilibrium endowment level

$$\begin{aligned} b_t &= \frac{2\gamma\mu(1+r)^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}} [r\omega(2\mu+1)^2 - 2\gamma\mu]}{r(2\mu+1)^2 \left[ (1+r)^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}}(r-\rho+1) \right]} \\ \omega_t &= \frac{\left[ (1+r)^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}} \right] \{ r [2\gamma\mu + \omega(2\mu+1)^2] - 2\gamma\mu\rho \}}{r(2\mu+1)^2 \left[ (1+r)^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}}(r-\rho+1) \right]}. \end{aligned} \quad (12)$$

Note that  $b_t > 0$  and  $\omega_t > 0$  by Assumption 4.

Substituting (8), (9), and (12) into the consumption resource constraints specified in (4) and (5), the steady state equilibrium consumption paths for generations  $t = 0, 1, 2, \dots$  are

$$c_t^y = \frac{\{ r [2\gamma\mu + \omega(2\mu+1)^2] - 2\gamma\mu\rho \} (1+r)^{\frac{\theta-1}{\theta}}}{r(2\mu+1)^2 \left[ (1+r)^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}}(r-\rho+1) \right]} \quad \text{and} \quad c_{t+1}^o = [\delta(1+r)]^{\frac{1}{\theta}} c_t^y. \quad (13)$$

Note that the steady state consumption of the initial old in period  $t = 0$  (generation  $t = -1$ ) is only  $c_0^o$ .

In general, utility maximization implies that there should be some intertemporal consumption smoothing. From (13) we can conclude that the equilibrium exhibits the property that old-age consumption is proportional to young-age consumption. That is,  $c_{t+1}^o/c_t^y = [\delta(1+r)]^{\frac{1}{\theta}}$ . Therefore, although changes in the degree of common ownership  $\mu$  affect the return of share acquisition relative to the return on bonds, consumers always readjust their consumption levels to maintain a fixed ratio of  $[\delta(1+r)]^{\frac{1}{\theta}}$  between the two periods.

## 5. Effects of common ownership on real investment, consumption, and welfare

Each generation of young consumers can channel resources into the acquisition of shares of institutional investors  $v_t$ , and to bond-financed interest-bearing real investment valued at  $b_t$ . According to Definition 1, the share acquisition constitutes a financial investment in the sense that it functions as a transfer of ownership of dividend-paying assets  $v_t$  from the old generation to the young one, and in this sense it does not yield a real return. In contrast,  $b_t$  constitutes a real investment as it enhances the endowment of the next generation, thereby yielding a social real return  $\rho$ .

This section analyzes the effects of an increased degree of common ownership, captured by the parameter  $\mu$ , on young consumers' allocations of savings and lifetime consumption. The young consumer determines the optimal allocation *after* receiving the period  $t$  endowment, as specified in (12) and (13).

### 5.1 The effects of common ownership on real investment

The equilibrium amount of real investment in each period  $t$  is given in (12). Differentiating (12) with respect to  $\mu$  yields

$$\frac{\partial b_t}{\partial \mu} = \frac{2\gamma(2\mu - 1) \left[ (1 + r)^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}} \right]}{r(2\mu + 1)^3 \left[ (1 + r)^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}}(r - \rho + 1) \right]} > 0 \quad (14)$$

by Assumptions 1 and 4. From (14) we can draw the following conclusion.

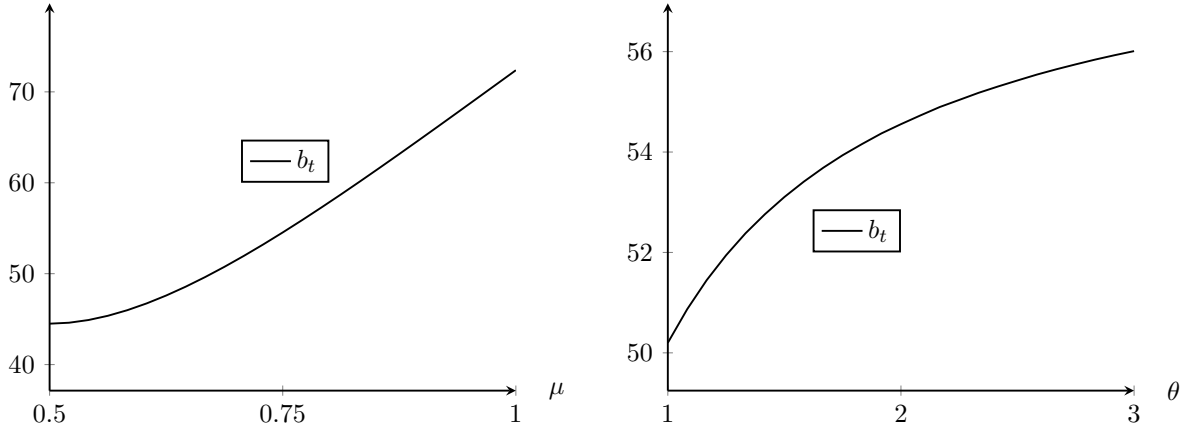
**Result 2.** *The young consumers in each period  $t$  allocate more of their savings into bond-financed real investments when the institutional investors maintain a lower degree of common ownership in the producing firms (higher  $\mu$ ). Formally,  $\partial b_t / \partial \mu > 0$ .*

The left panel of Figure 2 illustrates Result 2 for all admissible values of  $\mu$  by plotting  $b_t$  given in (12) for some parameter values. It shows that real investment rises with higher shares of majority ownership ( $\mu$  increases from 50 to 100 percent). Result 2 implies that a higher degree of common ownership (lower  $\mu$ ) reduces bond-financed real investments. In particular, real investments are minimized when the institutional investors co-own the competing producing firms with equal ownership shares ( $\mu = \frac{1}{2}$ ).

The right panel of Figure 2 demonstrates that the real investment increases as a function of the relative risk aversion. We comment on this property in subsection 5.4.

The intuition behind Result 2 is as follows. Result 1(a) shows that competition between the producers is intensified when each institutional investor maintains a higher majority share in one of the producing firms (higher  $\mu$ ). This reduces equilibrium price and profits. Therefore, a lower degree of common ownership (higher  $\mu$ ) reduces dividends, thereby making the financial investment less attractive to young consumers. This induces young consumers to direct a higher portion





**Figure 2:** Equilibrium real investment as function of the rate of institutional investors' majority ownership  $\mu$ , and relative risk aversion parameter  $\theta$ . *Note:* Both graphs are based on  $\gamma = 49$ ,  $\delta = 0.9$ ,  $\rho = 0.1$ ,  $r = 0.05$ , and  $\omega = 600$ . The left panel is based on  $\theta = 2$ . The right panel on  $\mu = 0.75$ . Observe that  $b_t = 54.55$  on both panels when  $\mu = 0.75$  and  $\theta = 2$ .

of their savings into bond-financed real investments.

An alternative interpretation of Result 2 is as follows. Result 1(b) states that profits reach the highest level with diversified ownership such that the institutional investors hold equal ownership shares in the competing firms ( $\mu = \frac{1}{2}$ ). This means that common ownership is a device for weakening product market competition thereby strengthening institutional investors' market power. Market power due to diversified ownership then translates into higher market value of institutional investors' ownership shares and higher dividends. This, in turn, implies that the acquisition of institutional ownership requires more resources from the young generation, and these resources have to be diverted from the bond-financed real investments. Therefore, a higher degree of common ownership by the institutional investors (as captured by lower values of  $\mu$ ) "crowds-out" real investment. A similar feature of overlapping generations models has been established before within the framework of different applications, and it is a direct consequence of the liquidity constraint imposed on young consumers.<sup>4</sup>

<sup>4</sup>This type of crowding-out effect in an OLG model was first identified in Laitner (1982), where he demonstrates how imperfect competition affects aggregate output and capital accumulation. Subsequently, Chou and Shy (1991, 1993), Jones and Manuelli (1992), and Shy and Stenbacka (2019a) apply this finding to analyze distortions caused by long duration of patents, insufficient investment and growth, and traditional banking, respectively.

## 5.2 The effects of common ownership on consumption

The equilibrium consumption levels of the young and old agents in each period  $t$  are given in (13).

Differentiating (13) with respect to  $\mu$  yields

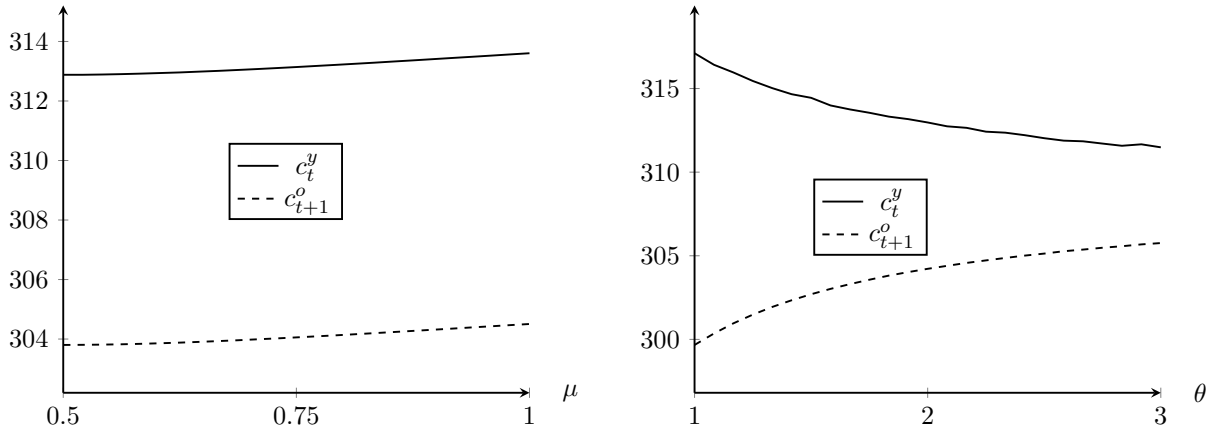
$$\frac{\partial c_t^y}{\partial \mu} = \frac{2\gamma(2\mu - 1)(\rho - r)(1 + r)^{\frac{\theta-1}{\theta}}}{r(2\mu + 1)^3 \left[ (1 + r)^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}}(r - \rho + 1) \right]} > 0 \quad (15)$$

by Assumptions 1 and 4. We summarize our findings in the following result.<sup>5</sup>

**Result 3.** *An increase in the degree of common ownership held by the institutional investors (lower  $\mu$ ) will reduce the consumption of young and old consumers. Formally,  $\partial c_t^y / \partial \mu > 0$  and  $\partial c_{t+1}^o / \partial \mu > 0$ .*

The last part follows from (13) which shows that the equilibrium consumption of the old is proportional to the consumption of the young.

The left panel of Figure 3 illustrates Result 3 for all admissible values of  $\mu$  by plotting  $c_t^y$  and  $c_{t+1}^o$  given in (13) for some parameter values. The right panel of Figure 3 shows young-age as well as old-age consumption as a function of the relative risk aversion, and we return to comment on this in subsection 5.4.



**Figure 3:** Equilibrium consumption as functions of the rate of institutional investors' majority ownership  $\mu$  and the utility parameter  $\theta$ . *Note:* Both graphs are based on  $\gamma = 49$ ,  $\delta = 0.9$ ,  $\rho = 0.1$ ,  $r = 0.05$ , and  $\omega = 600$ . The left panel is based on  $\theta = 2$ . The right panel on  $\mu = 0.75$ . Observe that  $c_t^y = 312.97$  and  $c_{t+1}^o = 304.24$  on both panels when  $\mu = 0.75$  and  $\theta = 2$ .

<sup>5</sup>Result 3 as well as Result 4 are restricted to comparisons among steady-state equilibria. Therefore, the results do not necessarily apply to a change  $\mu$  occurring in a certain period, such as  $t = 0$ . In this case, the consumption of the old at  $t = 0$  (generation  $t = -1$ ) will decrease with an increase in  $\mu$  because of a decline in the value of the shares sold by the old to the young generation in  $t = 0$ .

Result 3 is central to our analysis as it highlights that both young and old consumers in each period lose from an increase in institutional investors' degree of common ownership in the product market firms. The reason is that a higher degree of common ownership ( $\mu$  decreases towards  $\frac{1}{2}$ ) raises the value of the ownership shares in the institutional investors. This induces the young to divert savings from bond-financed real investments to financial investments, which results in lower resource endowments, and hence lower consumption in a steady state equilibrium.

### 5.3 The effects of common ownership on consumer welfare

This paper analyzes two separate, but related, markets. Section 2 analyzes the product market with two firms owned by institutional investors. Result 1 shows that a higher degree of common ownership by institutional owners corresponds to a higher market price. Hence, it is detrimental for consumer welfare. The reason is that a higher degree of common ownership increases firms' market power. All subsequent sections, the core of our analysis, build on Section 2 but focus on consumers in their role as owners of institutional investors with ownership of the product market firms. In particular, these sections characterize how the profits earned by institutional investors from owning the producing firms generate financial value, which forms the basis of assets that can be traded among generations. The trade of ownership of institutional investors importantly impact on how agents with finite horizons intertemporally allocate their resources between consumption, ownership of institutional investors, and bond-financed real investments.

As shown in (15) (Result 3) and as illustrated in the right panel of Figure 2, an increase in  $\mu$  raises the consumption of each generation, young-age as well as old-age consumption. By the monotonicity property of a generation's lifetime utility function (6), we can immediately conclude that the welfare of each generation of consumers rises with an increase in  $\mu$ . Thus, consumer welfare decreases with the degree of common ownership. Therefore, we can conclude that an increase in the degree of common ownership hurts consumer welfare independently of whether we focus on consumers from a traditional product market perspective or whether we focus on consumers as owners of institutional investors with ownership of the firms in the product market.

We conclude our analysis by formulating the following result.

**Result 4.** *An increase in the degree of common ownership  $\mu$  is welfare reducing.*

It should be emphasized that Result 4 provides a comparison among different steady state equilibria. This comparison is different from an analysis of a regime change (change in  $\mu$ ) at a particular period (say,  $t = 0$ ).<sup>6</sup>

Result 4 shows that higher degrees of common ownership are inefficient. Static oligopoly models in industrial economics typically emphasize that common ownership weakens competition and therefore tends to hurt consumers. Such a perspective focuses on consumption opportunities restricted to the industry under consideration. This paper analyzes consumers in their role as shareholders of institutional owners, say pension funds, under circumstances where these individuals have consumption opportunities outside the industry under consideration. Thus, the present analysis focuses on consumers in the role of investors seeking to optimize the net present value of lifetime consumption.<sup>7</sup>

According to Result 4, common ownership by institutional investors of firms selling in the same market not only weakens product market competition, but also harms generations of consumers by diverting their savings from real investments to acquisition of more costly shares of institutional investors. In this respect, common ownership by institutional investors that own firms competing in the same market leads to a distortion in consumers' intertemporal resource allocations. This distortion augments the traditional welfare loss familiar from static oligopoly models capturing cross-ownership.

## 5.4 Risk aversion and consumption smoothing

Figure 2 illustrates that real investment increases with the degree of relative risk aversion ( $\theta$ ) in the utility function. In Appendix C we show that  $\delta(1+r) < 1$  is a sufficient condition for this to hold in general. A higher degree of relative risk aversion implies that the utility function is more concave. This increases consumers' benefits from increased intertemporal income smoothing, which can be

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<sup>6</sup>In view of Footnote 5, a decrease in  $\mu$  in a given period, such as  $t = 0$ , will increase the welfare of the old in period  $t = 0$  but may decrease that of all subsequent generations. In this case, the consumption allocation after the change in  $\mu$  will not be Pareto comparable to the allocation before the change in  $\mu$  is made.

<sup>7</sup>Merton and Thakor (Forthcoming) explore some implications associated with the dual role of individuals as both customers and investors of financial institutions.

achieved by increasing the real investment  $b_t$  as an instrument to transfer some consumption from young to old age.

The right panel in Figure 3 verifies the same intuition. It shows that a higher degree of relative risk aversion leads to stronger incentives for intertemporal consumption smoothing, whereby agents sacrifice some young-age consumption in order to facilitate higher old-age consumption. We capture this feature analytically by the calculations presented in Appendix C.

## 6. Uncertain returns

The analysis so far has been based on the assumption with no uncertainty regarding the gross return on real investment  $\rho$  as well as the interest paid on bonds financing this project  $r$ . Therefore, the endowment expansion rule (9) implies that a period  $t$  bond-financed real investment  $b_t$  enhances the period  $t + 1$  endowment to  $\omega_t = \omega + b_t(\rho - r)$  with certainty. This section sketches an extension of the model to incorporate uncertain returns.

Suppose now that  $\tilde{\rho}$  and  $\tilde{r}$  are random variables, taking values  $\tilde{\rho} = \rho$  and  $\tilde{r} = r$  with probability  $\sigma$  (success), where  $0 < \sigma \leq 1$ , and  $\tilde{\rho} = \tilde{r} = 0$  with probability  $1 - \sigma$ . Otherwise,  $\tilde{\rho} = \tilde{r} = 0$  with probability  $1 - \sigma$ . The introduction of uncertain returns has two direct effects: The first effect is to turn the period  $t + 1$  endowment into a random variable for any given period  $t$  real investment  $b_t$ . Formally,

$$\tilde{\omega}_{t+1} = \begin{cases} \omega + b_t(\rho - r) & \text{probability } \sigma \\ \omega & \text{probability } 1 - \sigma. \end{cases} \quad (16)$$

The second effect would make the period  $t + 1$  return on generation  $t$ 's real investment into a random variable taking values  $b_t(1 + r)$  with probability  $\sigma$  and  $b_t$  with probability  $1 - \sigma$ .

Consider an arbitrary generation  $\tau$ . The two effects of uncertainty described above are viewed differently from the perspective of the young generation in period  $\tau$ . The endowment  $\omega_\tau$  received by the young in  $\tau$  is not random, but is given at  $\bar{\omega}_\tau$ , which takes one of the two values given in (16). That is, the young collect their endowment after the return on the real investment made in  $\tau - 1$  is realized according to (16). In contrast, the young in  $\tau$  do face uncertainty regarding the period  $\tau + 1$  return on their real investment  $b_\tau$ .

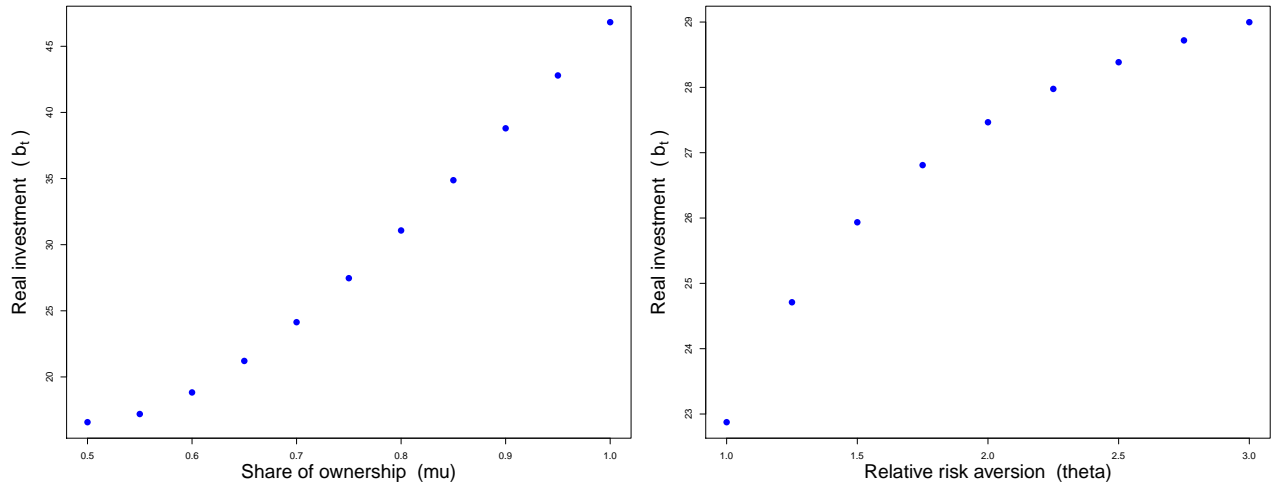
For a given endowment  $\bar{\omega}_t$ , the optimization problem (6) facing the young generation is now

generalized to

$$\begin{aligned} \max_{b_t} E_t U^t &= \frac{1}{1-\theta} \left[ \bar{\omega}_t - \frac{2\gamma\mu}{\sigma r(2\mu+1)^2} - b_t \right]^{1-\theta} \\ &+ \frac{\delta\sigma}{1-\theta} \left[ \frac{2\gamma\mu}{(2\mu+1)^2} + \frac{2\gamma\mu}{\sigma r(2\mu+1)^2} + b_t(1+r) \right]^{1-\theta} \\ &+ \frac{\delta(1-\sigma)}{1-\theta} \left[ \frac{2\gamma\mu}{(2\mu+1)^2} + \frac{2\gamma\mu}{\sigma r(2\mu+1)^2} + b_t \right]^{1-\theta}, \end{aligned} \quad (17)$$

where  $E_t$  captures the expectation formed in period  $t$  regarding the investment return in period  $t+1$ . Because the expected return on the real investment is now  $\sigma r$  (instead of  $r$ ), the discounted expected value  $v_t$  in (8) is now modified to  $v_t = \frac{2\gamma\mu}{\sigma r(2\mu+1)^2}$ . This appears as the second term in each row in (17). Note that the expected utility function (17) is continuous with respect to the success probability in the sense that (17) converges to the certainty case (11) as  $\sigma \rightarrow 1$ .

Appendix D derives the first-order condition for optimization problem (17) regarding the real investment. It is impossible to present closed form-solutions to this first-condition. Instead, Figure 4 uses numerical methods to mimic the simulations displayed in Figure 2.<sup>8</sup>



**Figure 4:** Numerical simulations of real investment as function of the rate of institutional investors' majority ownership  $\mu$ , and the relative risk aversion parameter  $\theta$ . *Note:* Both graphs are based on  $\gamma = 49$ ,  $\delta = 0.9$ ,  $\rho = 0.1$ ,  $r = 0.05$ , and  $\omega_t = 600$ . The left panel is based on  $\theta = 2$ . The right panel on  $\mu = 0.75$ . Observe that  $b_t = 27.46$  on both panels when  $\mu = 0.75$  and  $\theta = 2$ .

Comparing Figure 4 with Figure 2 confirms the same monotonic relationships between real

<sup>8</sup>The graphs are drawn in R using the `uniroot` function to extract  $b_t$  from the implicit first-order condition (D.1).

investment  $b_t$  and the share of ownership  $\mu$ , as well as between real investment and the relative risk aversion parameter  $\theta$ . We would like to caution the reader that despite the similarity between the two figures, this section with uncertainty provides only a partial solution to the real investment optimization as the investment-inclusive endowment is taken as a constant. In contrast, Figure 2 is based on a complete solution where the steady-state investment-inclusive endowment  $\omega_{t+1}$  is allowed to vary with  $b_t$ . This explains why the calibrated values of  $b_t$  in Figure 4 are slightly different from those in Figure 2.

Finally, to obtain strictly positive values of  $b_t$ , the numerical analysis relied on success probability  $\sigma = 0.9$ . These simulations show that there is no real investment for success rates lower than  $\sigma = 0.8$  under the parameter values listed in Figure 4

## 7. Conclusion

This analysis shows that an increased degree of common ownership by institutional investors that own producers competing in the same product market leads to a reduction in real investments. In a steady state equilibrium, this implies a reduction in the consumption of the young as well as old consumers. Overall, common ownership of competing producers by institutional investors may serve as a device that not only weakens product market competition, but also harms consumers by making it more beneficial for them to divert resources from real investments to acquisition of more expensive ownership of institutional investors.

According to the well-known Schumpeterian view (for surveys, see Kamien and Schwartz (1982) or Martin (1993)), dynamic considerations associated with innovation incentives tend to reduce the harm to consumers from firms with market power. This is often expressed by arguing that market power of firms defines a tradeoff between dynamic and static efficiency. According to our analysis there is no such tradeoff between dynamic and static efficiency associated with common ownership by institutional investors in same-industry product market firms. The reason is that common ownership induces a distortion of consumers' resource allocation from real investments to financial investments (ownership), thereby reinforcing the familiar welfare loss associated with static models of cross-ownership between firms in oligopoly markets.

## Appendix A Algebraic derivations for Section 2

**Derivations of the static equilibrium values (3).** Substituting (1) into (2), the first-order conditions for (2) are

$$0 = \frac{\partial \pi_A}{\partial q_1} = \alpha\mu - 2q_1\beta\mu - q_2\beta \quad \text{and} \quad 0 = \frac{\partial \pi_B}{\partial q_2} = \alpha\mu - 2q_2\beta\mu - q_1\beta. \quad (\text{A.1})$$

The second-order conditions are  $\partial^2 \pi_A / \partial (q_1)^2 = \partial^2 \pi_B / \partial (q_2)^2 = -2\beta\mu < 0$ . Solving the system of two first-order conditions (A.1) for  $q_1$  and  $q_2$ , and then substituting into the inverse demand function and the profits (2) yield (3).

**Derivation of Result 1.** To prove part (a), differentiating (3) yields  $\partial p / \partial \mu = -2\alpha / (2\mu + 1)^2 < 0$  and  $\partial \pi_A / \partial \mu = \partial \pi_B / \partial \mu = \alpha^2(1 - 2\mu) / [\beta(2\mu + 1)^3] < 0$  because  $\mu < 1/2$ . Part (b) follows from part (a) noting that  $\mu = 1/2$  is the lowest possible value under Assumption 1.

## Appendix B Algebraic derivations for Section 4

**Derivations of the steady state equilibrium (12).** The the first-order condition for the lifetime utility maximization problem (11) is

$$0 = \frac{\partial U^t}{\partial b_t} = \delta(1 + r) \left\{ \frac{(1 + r) [b_t r(2\mu + 1)^2 + 2\gamma\mu]}{r(2\mu + 1)^2} \right\}^{-\theta} - \left[ -\frac{b_t r(2\mu + 1)^2 - r\omega_t(2\mu + 1)^2 + 2\gamma\mu}{r(2\mu + 1)^2} \right]^{-\theta}. \quad (\text{B.1})$$

$b_t$  is extracted from (B.1) to obtain (12). The second-order condition is

$$\frac{\partial^2 U^t}{\partial (b_t)^2} = -\theta \left[ \frac{(2\mu + 1)^2 r(\omega_t - b_t) + 2\gamma\mu}{r(2\mu + 1)^2} \right]^{-\theta-1} - \left[ \frac{r\delta\theta(r + 1)(2\mu + 1)^{2(\theta+1)} r^\theta (r + 1) [b_t r(2\mu + 1)^2 + 2\gamma\mu]^{-\theta}}{b_t r(2\mu + 1)^2 + 2\gamma\mu} \right] < 0. \quad (\text{B.2})$$



## Appendix C Algebraic derivations for Section 5

**Derivations of the slope in Figure 2 (right panel)** Differentiating (12) with respect to  $\theta$  yields

$$\frac{\partial b_t}{\partial \theta} = \frac{\delta^{\frac{1}{\theta}} \{r [2\gamma\mu + \omega(2\mu + 1)^2] - 2\gamma\mu\rho\} (1+r)^{\frac{\theta-1}{\theta}} \ln \left[ \frac{1}{\delta(1+r)} \right]}{r\theta^2(2\mu + 1)^2 \left[ (1+r)^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}}(r - \rho + 1) \right]^2}. \quad (\text{C.1})$$

The denominator is positive. The first term in the numerator is positive by Assumption 4(a). The logarithmic factor in the numerator is positive provided that  $\delta(1+r) < 1$ , which would then becomes a sufficient condition for  $\partial b_t / \partial \theta > 0$ .

**Derivations of the slope in Figure 3 (right panel)** Differentiating (13) with respect to  $\theta$  yields

$$\frac{\partial c_t^y}{\partial \theta} = \frac{\delta^{\frac{1}{\theta}}(r - \rho + 1) \{2\gamma\mu\rho - r [2\gamma\mu + \omega(2\mu + 1)^2]\} (1+r)^{\frac{\theta-1}{\theta}} \ln \left[ \frac{1}{\delta(1+r)} \right]}{r\theta^2(2\mu + 1)^2 \left[ (1+r)^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}}(r - \rho + 1) \right]^2}. \quad (\text{C.2})$$

The term in the curly brackets is negative by Assumption 4(a), whereas the factor  $r - \rho + 1$  in the numerator is positive by Assumption 4(b). All other terms are identical to (C.1), meaning that  $\delta(1+r) < 1$  is a sufficient condition for  $\frac{\partial c_t^y}{\partial \theta} < 0$ . Finally,

$$\frac{\partial c_{t+1}^o}{\partial \theta} = \frac{\delta^{\frac{1}{\theta}} \{r [2\gamma\mu + \omega(2\mu + 1)^2] - 2\gamma\mu\rho\} (1+r)^{\frac{1-2\theta}{\theta}} \ln \left[ \frac{1}{\delta(1+r)} \right]}{r\theta^2(2\mu + 1)^2 \left[ (1+r)^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}}(r - \rho + 1) \right]^2}. \quad (\text{C.3})$$

The term in the curly brackets is positive by Assumption 4(a), and all other terms are also positive as long as  $\delta(1+r) < 1$ . We can therefore conclude that  $\frac{\partial c_{t+1}^o}{\partial \theta} > 0$ .

## Appendix D Algebraic derivations for Section 6

Differentiating (17) with respect to  $b_t$  yields

$$\begin{aligned} 0 = \frac{\partial EU^t}{\partial b_t} &= \delta\sigma^{\theta+1}(1+r) \left[ \frac{b_t r \sigma (1+r)(2\mu + 1)^2 + 2\gamma\mu(r\sigma + 1)}{r(2\mu + 1)^2} \right]^{-\theta} \\ &\quad - \left[ -\frac{b_t r \sigma (2\mu + 1)^2 - r\sigma\omega(2\mu + 1)^2 + 2\gamma\mu}{r\sigma(2\mu + 1)^2} \right]^{-\theta} \\ &\quad + \delta(1-\sigma) \left[ \frac{b_t r \sigma (2\mu + 1)^2 + 2\gamma\mu(r\sigma + 1)}{r\sigma(2\mu + 1)^2} \right]^{-\theta}. \end{aligned} \quad (\text{D.1})$$

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