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Parties as efficiency-improving gate-keepers in rent-seeking societies

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Abstract

Anti-corruption laws forbid selling public-job nominations. Even if bribing is ruled out, those interested in the nominations may invest in good relationships with the nominators. This provides a legal way to influence the decision. Such networking is costly, however. Thus, rent-seeking results in excessive networking.

We present a simple model featuring such effects and show that efficiency may be improved if political parties interfere with the nominations. Political parties may reduce wasteful networking, thanks to exclusive membership contracts. Parties can require that politicians belonging to the party promote the nomination of other party members, thus, reducing incentives to cultivate inter-party connections.

Keywords: Political parties, Political Nominations, Rent-seeking, Connections, Networks

JEL classification: D72, D85, L14
1 Introduction

Politicians have influence on a variety of nominations. Occasionally when a nomination is decided upon, a politician finds himself in a pivotal position. Anti-corruption laws forbid the politician to sell the nomination. Still, even though interested citizens can not buy a position, it pays off to be on good terms with politicians. Citizens need connections to the nominating politicians to have a chance to be nominated. Keeping in touch takes time from both citizens and politicians. Yet, gains are distributed asymmetrically: citizens benefit from a chance to be nominated, while politicians have to spend their time on networking. So why would politicians spend time with rent-seekers? Miettinen and Poutvaara (2014) suggest that when anti-corruption laws bind, rent-seekers can pay to access decision-makers, even when anti-corruption laws rule out selling the spoils that politicians distribute. The citizens spend time with the politicians by offering lunches and entertainment, and by taking part in campaigns and fund-raising events to be remembered when a nomination is made. In addition, if there are no restrictions on whom the politician can nominate, citizens gain by rubbing shoulders with several politicians. This is excessively time-consuming and results in wasteful networking.\footnote{Additional links provide no efficiency gains, at least in a world of equally capable candidates who are equally valuable when nominated.}

Here, political parties may provide valuable services.

Political parties are powerful gatekeepers in modern democracies. Citizens looking for spoils, and politicians allocating these, cannot belong to more than one party at a time. Moreover, party membership is divided into subgroups with prominent politicians as leaders. Examples of such subgroups include local party associations, as well as party associations for women, young people, students or pensioners. For the welfare consequences of rent-seeking through connections, it is crucial whether and to what extent parties are allowed to act as gatekeepers requiring their politician members to nominate other party members or members belonging to their group. If politicians are expected to favor only members belonging to their group, then party members outside this group, let alone members of other parties, have no incentive to lobby this politician. The parties can thus reduce rent-seeking. It is important to note that the role of political parties need not be that of an intermediary meddling in the allocation of actual nominations. All that is needed is a gate-keeping role introducing a mechanism of directing politicians to nominate party members from some sub-group only. This would be easiest to implement in one-member districts: the party could simply direct politicians to nominate rent-seekers who reside in their district (rent-seekers could also network with their party’s candidate before the election with the winning party’s candidate then being in a position to fill a job after the election). Depending on the spoils distributed, nominations may take place at different levels. For example, in the United States, senators play a gate-keeping role for nominations of federal judges to their state, while many other jobs are filled at local level. The identity
of the party boss and the politician could then vary depending on the context. Similar mechanisms could be at play at local level, county level, state level or federal level, depending on the spoils being distributed. In Western Europe, it is quite common that certain important nominations are earmarked to representatives of a certain political party, both at national and local level. Perhaps the most extreme case is Austria, where it has not been uncommon that public organizations have double heads - one for Social Democrats, one for Conservatives. Political parties are even involved in the choice of directors in public companies, including but not limited to television and radio, and in nominations to jobs at school and local public administration (Encyclopedia of Austria, 2005; Profil, 2013).

This paper formalizes the argument presented above and shows that political parties may play the gatekeeping role that improves efficiency if we assume that citizens are equally valuable when nominated. We compare rent-seeking through connections with and without the role of the political parties. We take as our starting point that political parties exist and that the politicians decide upon the allocation of non-ideological rents. We illustrate that allowing the political parties to play the gate-keeping role improves efficiency in the distribution of such rents.

Even if we find that parties’ involvement in the nominations may increase welfare, this need not always hold. This is because there are additional costs of networking when the parties are present: there is need for additional within-party networking when the party leadership coordinates and keeps in touch with the politicians. Of course, this latter concern does not arise if the connections between party leadership and the politicians exist, whether or not the party is involved in political nominations. In this latter case the parties’ interference with nominations is unambiguously welfare-enhancing.

Our model abstracts from the ideological considerations. This is not because we consider ideology unimportant. Rather, we abstract from the ideology to uncover the full potential of rent-seeking in explaining the role of political parties in non-ideological nominations. The effects of including ideological considerations are briefly discussed in the conclusion.

The paper is organized as follows. Section 2 summarizes related literature. Section 3 presents the model when parties are not involved. This section is based on the results of Miettinen and Poutvaara (2014) which studies properties of a stable rent-seeking network absent any gate-keeping intermediary. Section 4 contains novel results regarding the gate-keeping role of parties. Section 5 presents the welfare comparisons between the regimes. Section 6 concludes.
2 Related literature

Our analysis has common features with several strands of literature. First, our approach is related to the literature on rent-seeking and lobbying contests (Tullock 1967, 1980; Hillman and Katz 1984; Bernheim and Whinston, 1986; Grossman and Helpman, 1994; Konrad, 2000; Franke, 2012; Amegashie 2012; see Konrad, 2009 and Long 2013 for recent overviews) which gains important insights into how lobbying may affect policy-making. These models are similar to our model in that citizens actively influence the politicians’ decisions on how to distribute rents. Yet, there are major differences. In our model, links are endogenous, requiring mutual consent. Moreover, the links are costly not only for the lobbying side but also for the politicians. Payments are made in exchange for establishing links. In the rent-seeking and lobbying literature, the links are given at the outset. Second, only the citizens bear costs. Third, costs are bids in an auction or in a contest.

Throughout the analysis, we assume that anti-corruption laws work and thus the nominations cannot be auctioned or traded even implicitly. Therefore, we have especially in mind a modern democracy with a relatively low level of corruption such as EU 15 and especially Nordic countries. Previous literature on contests has already extensively analyzed the case where the anti-corruption laws can be circumvented.

The only previous contributions that endogenize the relationship between politicians and lobbyists are Felli and Merlo (2006) and Miettinen and Poutvaara (2014). In those papers, there are no intermediaries or gatekeepers; in the present paper we compare outcomes without intermediaries to those in the presence of parties as gatekeepers. Moreover, Felli and Merlo analyze ideological lobbying and assume that the links are costless. We analyze lobbying on non-ideological spoils with costly networking.

Second, our analysis is related to the middlemen literature (Rubinstein and Wolinsky, 1987) and the literature on two-sided markets (Rochet and Tirole, 2008). The previous literature has focused on situations in which the intermediary or the middleman facilitates search and matching. We analyze the case where the intermediary, the political party, plays a useful role by restricting activity between the two sides of its market. Therefore, the intermediary effectively serves as gatekeeper in the establishment of links, but does not intervene in the actual rent allocation.

Third and finally, our explanation complements previous efficiency rationales for the prominent role of political parties, like Alesina (1988), Alesina and Spear (1988) and Caillaud and Tirole (2002). These previous contributions do not touch upon the

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2The inability of politicians to sell or to auction off nominations when these arise could result from outside monitoring or from there being a fraction of honest citizens and politicians who would report asking or offering bribes, provided that punishments for corruption are sufficiently high.

3See the Transparency International Corruption Perceptions Index (2013).

4See also Köppel-Turyna (2014) who studies empirically how restrictions on private campaign contributions influence the ideological positioning of party platforms.
question why political parties play a role also in cases where they do not reduce the time spent searching, provide additional information, or solve various commitment problems. It is questionable to what extent a political party would provide new information when filling the positions of trust or public jobs in a small municipality, for example. Yet, these positions and many other jobs are typically earmarked to different political parties. Our explanation for the role of a political party applies also in these cases.

3 Equilibrium without political parties

3.1 The model

The setup of the model in this section coincides with that in Miettinen and Poutvaara (2014) where the proof of Proposition 1 below can also be found. Thus, the analysis is presented fairly briefly.

In this section, we assume that the political parties do not meddle in nominations. There are two types of agents. Type $A$ is called a politician and type $B$, someone interested in being nominated, a citizen. Each politician makes a nomination with probability $p$. A nominated citizen receives surplus $s$ where $s$ is strictly positive. We define the expected rent as $\psi = ps$.

There are $n_A$ politicians and $n_B$ citizens. The politicians are indexed with $i = 1, \ldots, n_A$ and the citizens with $j = 1, \ldots, n_B$. There are $\gamma$ times more citizens than politicians, $n_B = \gamma n_A$, where $\gamma$ is an integer strictly greater than one.

Whether politician $i$ is connected with citizen $j$ is captured by $m_{i,j}$. If $i$ is connected with $j$ then $m_{i,j} = 1$, if not then $m_{i,j} = 0$. A connection is established between a politician and a citizen if both are willing to do so. Politician $i$’s connections are described by $m_i^A = (m_{i,1}, \ldots, m_{i,n_B})$ and citizen $j$’s connections are described by $m_j^B = (m_{1,j}, \ldots, m_{n_A,j})$. Thus the network is characterized by the matrix

$$M = (m_1^B, \ldots, m_{n_B}^B) = (m_1^A, \ldots, m_{n_A}^A)'$$

The number of connections that citizen $j$ has to politicians is denoted by $m_{jA} = \sum_{i=1}^{n_A} m_{i,j}$ and the number of connections that politician $i$ has to citizens is denoted by $m_{iB} = \sum_{j=1}^{n_B} m_{i,j}$.

Maintaining a connection is costly. A decreasing marginal productivity in other activities, or a decreasing marginal utility of leisure, implies that the marginal cost of time spent on networking is strictly increasing. We adopt a simple quadratic cost for politician $i$ of having a total number $m_{iB}$ of connections, $\frac{c}{2} m_{iB}^2$. Similarly citizen $i$’s cost of networking is $\frac{c}{2} m_{iA}^2$. Here $c$ is a positive networking cost parameter independent of the agent’s type. Both must contribute time and effort to keep up the relationship. Apart from a convex opportunity cost of time being realistic, the quadratic cost function
implies that competition drives prices for links equal to marginal costs. In our model with homogeneous citizens and politicians, the link quantities will be (almost) uniquely defined and (almost) symmetric across agents of the same type. Moreover, the rents will not be entirely competed away (as in the case of constant marginal costs) but each agent receives a positive profit in equilibrium.

Politicians benefit from the connections by charging the citizens (implicit or explicit) rewards for keeping up a connection. We assume that, ex ante, each politician is indifferent as to whom to nominate and each citizen is indifferent as to who nominates him. However, we assume that in order for politician $i$ to be able to nominate citizen $j$, there has to be a direct connection between them, $m_{i,j} = 1$, as opposed to an indirect connection where $i$ knows a third agent who knows $j$. A politician nominates each citizen connected to her with an equal probability. Moreover, we make a simplifying assumption that a citizen can accept several nominations.$^5$

For a citizen, the probability that a connection to politician $i$ results in a nomination, $p_i$, depends negatively on the expected number of connections that the politician has to other citizens: the more connections to other citizens, the less likely it is that the politician nominates the citizen. To reflect the fact that the citizens cannot monitor the politicians, we assume that the citizens cannot observe how many other connections each politician is providing, not even ex post. (In fact a simple assumption of inability of politicians to commit not to sell additional links suffices to generate our results.) Yet, the citizens correctly anticipate the distribution of politicians' number of connections in equilibrium. Let the probability that politician has $k$ connections (or the fraction of politicians with $k$ connections) be denoted by $q_k^A$. By construction, the ex ante probability that a connection to a politician results in a nomination is, for all $i$, $p_i = p_A = \sum_k q_k^A \frac{p_k}{k}$. Similarly, the probability that citizen has $k$ connections is denoted by $q_k^B$.

Miettinen and Poutvaara (2014) build on the concept of \textit{pair-wise stability with transfers} developed by Bloch and Jackson (2007). This concept allows for monetary transfers being paid between the connecting parties which in our setup translate into payments from citizens to politicians in remuneration for their investment in keeping up the connection. A network is stable if no politician or citizen would gain by abolishing any of the specified connections or by quoting prices for alternative links and adding or replacing a connection. The formal statement is given in Miettinen and Poutvaara (2014) where we show using the definition that there is a pair-wise stable cooperative

$^5$Assuming alternatively that each citizen can only receive one nomination would have two effects. First, the probability of being offered a nomination would depend positively on the number of connections that other citizens (linked to the politician) have to other politicians. Second, the gain from an additional connection would not be constant but rather decreasing as with more connections to politicians, the probability that only one nomination is offered is decreasing. The politician’s incentives are unaffected by the alternative assumption, however, since she only cares about connections and rewards.
network with transfers which is unique up to permutations. Although the model does not assume away price discrimination, the market forces and the unobservability of connections\(^6\) implies that each politician charges each citizen the same reward \(r\). This reward is approximately equal to the marginal linking costs and equilibrates politicians’ supply and citizens’ demand for connections. This unique pair-wise stable network is symmetric: all citizens demand the same number and all politicians offer the same number of connections. Notice how the convex linking costs play a crucial role here.

In addition to paying the politicians \(r\) for maintaining the connections, the citizens have to pay their own linking costs. The expected payoff of citizen \(j\) when network \(M\) prevails with reward \(r\) reads

\[
E\pi_j(M, r) = m_{jA}p_A - m_{jA}r - \frac{c}{2}(m_{jA})^2
\]

and the payoff of politician \(i\) in the same network reads

\[
\pi_i(M, r) = m_{iB}r - \frac{c}{2}(m_{iB})^2.
\]

In the stable network, given rewards, increasing or decreasing the number of connections must not strictly pay off. Thus, we have the following condition for politicians

\[
\frac{c}{2}(2m_{AB} - 1) \leq r \leq \frac{c}{2}(2m_{AB} + 1).
\]

Each citizen takes as given the reward, \(r\), and correctly anticipates the expected probability of being nominated, \(p_A\). Again, in a stable network, all citizens behave identically and, given rewards, increasing or decreasing the number of connections must not strictly pay off:

\[
p_{AS} - \frac{c}{2}(2m_{BA} + 1) \leq r \leq p_{AS} - \frac{c}{2}(2m_{BA} - 1).
\]

We denote the total number of connections of a politician (to citizens) by \(m_{AB}^N\) and the total number of connections of a citizen (to politicians) in the stable network by \(m_{BA}^N\).\(^7\) Furthermore, we denote by \(r^N\) the uniform reward in the unique pairwise stable network.

In the stable network, the politicians demand the highest reward that the citizens are willing to pay given that they correctly anticipate the politician’s (expected) number of connections. Thus, one of the upper bounds of \(r^N\) in (3) and in (4) must be binding.

We denote the expected equilibrium payoffs by \(\pi_A^N, \pi_B^N\). The costs of networking are defined as

\[
TC^N = n_A \frac{c}{2}(q_{m_{AB}}^N (m_{AB}^N)^2 + (1 - q_{m_{AB}}^N)(m_{AB}^N + 1)^2) + n_B \frac{c}{2}(q_{m_{BA}}^N (m_{BA}^N)^2 + (1 - \]

\(^6\)Or the inability to commit not to sell additional links.

\(^7\)If politicians have two optima, this is the smaller of the link quantities that appear in the stable network. Due to symmetry and convexity of the linking costs, only two consecutive link quantities can be optimal.
where \( q_{m_{AB}B}^N ) ( m_{BA}^N + 1 )^2 \) where \( q_{m_{AB}A}^N \) and \( q_{m_{AB}B}^N \) are the probabilities, in a stable network, that a politician has \( m_{AB}^N \) connections and a citizen has \( m_{BA}^N \) connections, respectively. The sum of payoffs is defined by

\[
W^N = n_A \pi_A^N + n_B \pi_B^N.
\] (5)

The main proposition concerning the pairwise stable network without parties is a simple corollary of Proposition 1 of Miettinen and Poutvaara (2014). It is thus stated here without a proof.

**Proposition 1**

- There is a unique pair-wise stable network, provided that \( \psi \geq c \gamma^2 \). It is symmetric and associated with a uniform reward \( r^N \) charged by each politician from all citizens for each connection.

- In this network the numbers of connections, \( m_{AB}^N ( \psi, c, \gamma ) \) and \( m_{BA}^N ( \psi, c, \gamma ) \), are increasing in \( \psi \) and decreasing in \( c \) and in \( \gamma \).

- The payoffs, \( \pi_A^N, \pi_B^N \), the costs of networking, \( TC^N \), and the sum of payoffs, \( W^N \), are continuous and increasing in \( \psi \) but not necessarily strictly increasing.

The payoff functions are illustrated in Fig. 1 for the special case \( n_A = 2, \gamma = 2 \) and \( c = 1 \). As a function of \( \psi \), the citizen’s payoff is the curve on the bottom, the politician’s payoff is the second curve from the bottom and the aggregate surplus for two politicians and four citizens is the third curve from the bottom.

The total expected value of nominations is \( n_A \psi \). In Fig. 1, this is illustrated by the line starting from the origin with a slope equal to two. Notice, that the sum of payoffs falls short of this total expected value and the distance between these two increases in \( \psi \). The distance coincides with the total costs of networking.

### 4 Network with political parties

#### 4.1 The model

In this section, we introduce political parties or party bosses as gatekeepers between politicians and citizens. Party bosses are denoted by \( C \). The party bosses face the same networking costs as the politicians and the citizens. If party boss \( k \) has \( m_{kA} + m_{kB} \) connections to the politicians and the citizens respectively, then her cost of networking equals \( \frac{c}{2} ( m_{kA} + m_{kB} )^2 \). The bosses play an active role, making take-it-or-leave-it
Figure 1: Payoffs without parties.
offers to the politicians and to the citizens. There must be a direct connection between
the politician and the citizen who is nominated by the politician. To render the model
simple to solve, we assume that each party boss receives the right to control and design
the network of all the politicians and the citizens connected with her on the condition
that the party bears all the networking costs. However, the party bosses cannot commit
not to sign contracts with additional politicians and citizens. Each politician and
each citizen can join only one party at a time. The party boss maximizes the party’s
payoff.

For simplicity, the party bosses are exogenously given. To guarantee explicit sol-
lutions, we also assume that there are $\omega$ politicians per each party boss, where $\omega \in
\{2, 3, \ldots\}$. Therefore, $n_A = \omega n_C$ and as $n_B = \gamma n_A$ (by section 3), $n_B = \gamma \omega n_C$. The
party bosses are indexed by $k = 1, \ldots, n_C$. The party boss first recruits politician mem-
bers and then the citizen members. When the citizens make their networking decisions,
they know how many politicians belong to each party.

To reflect the empirical fact that politicians and their party bosses interact in various
ways, we assume that the politicians require having a direct connection with their party
boss. However, the party bosses need not interact directly with the citizens belonging
to the party, but they may delegate the member-contacts to the politicians representing
the party. There is an indirect connection between party boss $k$ and citizen $j$, when
there is a politician $i$ to whom both $k$ and $j$ are connected. We indicate an indirect
connection between $k$ and $j$ by $\mu_{k,j}$. If there is an indirect connection then $\mu_{k,j} = 1$,
if not, then $\mu_{k,j} = 0$. Once the party boss has made take-it-or-leave-it offers to the
politicians and the citizens and allocated citizens who have joined the party each to
a politician, the party boss no longer intervenes in the assignment of spoils from the
politicians to the citizens. Rather, citizens directly interact with the politicians from
whom they are expected to potentially receive spoils.

The service that the parties provide is the exclusivity of connections. Parties can
divide party members into groups around politicians: those belonging to the same elec-
toral district or demographic subgroup such as party associations for women, young
people, students or pensioners, for instance. If politicians are expected to nominate
only members belonging to their group, then party members outside this group, let
alone members of other parties, have no incentive to lobby this politician. This re-
duces wasteful excessive networking to each politician. Political parties are able to reap
part of the gains, as a reduction in the number of connections between politicians and

\begin{itemize}
  \item [8] If the party would not bear the networking costs, then the party boss would have an incentive to
  require politicians to build more connections \textit{ex post} than they have agreed on \textit{ex ante}.
  \item [9] We do not take a stance whether party bosses would keep the payoff, or part of it, for private
  consumption, or if they use the surplus for ideological purposes.
  \item [10] This timing would arise endogenously in a richer model in which party bosses decide the timing.
  If party bosses would not recruit politicians before selling connections to citizens, they might have
  an incentive to recruit fewer politicians \textit{ex post}. Thus, citizens would favor parties that have already
  secured connections to politicians.
\end{itemize}
rent-seekers increases the value of the remaining connections that are sold by political parties. Yet, there are costs to this as well since each politician must now have a connection to the party boss in addition to the citizens. There will be no direct payments between the politicians and the citizens since the party regulates all connections.\footnote{Note that citizens often pay the party in the form of volunteer work. Our framework could be generalized to allow for this, without changing the qualitative results.}

\section{Properties of the party network}

We focus on an equilibrium where all politicians and citizens are party members. We establish below conditions under which this is the case. For notational simplicity, the number of connections that an agent of type \( t \) has to \( t' \) types is set equal over the agents of the same type and denoted by the same variable for all agents of the same type. This is restrictive in general but we show in the appendix that this is a property of any equilibrium\footnote{This is essentially driven by the convexity of the networking cost function.}: all agents of the same type have an equal number of connections and pay and receive equal payments. These patterns are due to competition under the convex linking costs. Differences in link quantities across agents of a given type would imply that there are arbitrage opportunities at the margin and competitive forces would drive quantities equal again.

We denote the reward that each citizen pays to the party boss by \( r_{BC} \) and the reward that each politician receives from the party boss by \( r_{CA} \). A payment is made whether the connection is direct or indirect, but the cost of a connection is born only from direct connections. We also require that the equilibrium generates a non-negative payoff for all agents. We call these the constraints of political participation. It turns out that when these constraints are satisfied then both \( r_{BC} \) and \( r_{CA} \) are positive.

The party’s payoff is

\[
\pi_C = m_{CB}[r_{BC} - \frac{c}{2}(m_{BA} + m_{BC})^2] + \mu_{CB}[r_{BC} - \frac{c}{2}(m_{BA})^2] - m_{CA}[r_{CA} + \frac{c}{2}(m_{AB} + m_{AC})^2] - \frac{c}{2}(m_{CA} + m_{CB})^2.
\]

The first row is the sum of rewards paid by the citizens who are directly connected to the party net of the citizens’ networking costs paid by the party. The second row comprises the citizens who are indirectly connected to the party. The third row subtracts the rewards to the politicians and their networking costs paid by the party. The fourth row consists of the party’s own networking costs.

Equally, the citizen’s expected payoff is

\[
\pi_B = \frac{\psi}{m_{AB}}m_{BA} - r_{BC}.
\]
Finally, a politician connected to a party receives a reward from the party (in addition to the compensation for networking costs). Thus her payoff equals \( \pi_A = r_{CA} \).

The equilibrium network structure is based on two principles. First, the party is forced to build direct connections between the citizens and the politicians. This being the case, it is less costly for the party boss to establish her own connections to the citizens via a politician rather than directly. Moreover, allocating an equal number of citizens to each politician minimizes the cost of networking and maximizes the sum of citizens' willingnesses to pay. This is due to the convex linking costs and to the fact that the probability of being nominated decreases in the number of links that the politician has to citizens.

Second, competition drives the benefit from an additional connection equal to its marginal cost. Having a unique market reward and an unequal number of connections across the agents of one type would violate the condition of zero marginal net benefit. The one with fewer connections could apply the cheapest network structure described above and obtain the same reward with a lower marginal cost. Hence, the number of connections of any two agents of the same kind must be the same. The rewards are such that the parties are indifferent between supplying an additional connection to a citizen (via a politician), or demanding an additional connection to a politician, and sticking to the equilibrium number of connections.

The following proposition characterizes the network structure in the party equilibrium and establishes the existence conditions.

**Proposition 2**

- There is a party equilibrium with \( \pi_A^P \geq 0 \) and \( \pi_B^P \geq 0 \) where each citizen and each politician is connected to a unique party if and only if

\[
\frac{\psi}{c} \geq \gamma(\gamma + 2) \geq 2\omega + 2 + \gamma. \tag{6}
\]

- In this equilibrium, each party boss is connected to \( \omega \) politicians and \( \omega\gamma \) citizens. The numbers of direct connections are given by \( m_{AB} = \gamma, m_{BA}^P = 1, m_{AC} = 1, m_{CA}^P = \omega, m_{BC}^P = 0 = m_{CB}^P \).

**Proof.** See the appendix.

Competition drives the benefit from an additional connection equal to its marginal cost and the number of connections equal across the agents of one type. This implies that the reward that a citizen pays equals the marginal cost for the party boss of serving this additional citizen (the additional linking cost of the politician that now establishes a link to this citizen and the cost of bearing the citizen’s linking cost):

\[
r_{BC} = \left[\frac{c}{2}(\gamma + 2)^2 - \frac{c}{2}(\gamma + 1)^2 + \frac{c}{2}\right].
\]

Thus the citizen’s payoff equals

\[
\frac{\psi}{\gamma} - \frac{c}{2}(2\gamma + 4)
\]
which is non-negative if and only if the first inequality in (6) holds. Similarly the compensation paid to the politician by the party reflects the between-party competitive pressure that sets the reward equal to the net marginal benefit that an extra politician generates. The second inequality in (6) requires that this reward is non-negative. The term proportional to $\gamma c(\gamma + 2)$ on the left-hand side of this inequality reflects the rewards collected from the citizens linked to the (marginal) politician. The term proportional to $c(2\omega + 2 + \gamma)$ reflects the marginal costs that accrue to the party for establishing the network around the marginal politician: the party boss’s own marginal cost of linking to this additional politician and the total linking costs of this marginal politician and the citizens connected to this latter. The coefficient $c$ appearing on both sides can be thus ignored in this latter positive reward condition.

Notice that each type of agent (generically) earns a positive equilibrium payoff. Although “price” competition is fierce and renders rewards equal to marginal costs, the fact that cost functions are convex leaves positive profit for the inframarginal links. This is true both in the no-party equilibrium and in the party equilibrium. The service that the party provides to the politicians is the power of committing to lower number of links thus increasing the citizens’ willingness to pay for the connection to a party. Nonetheless, also the party is unable to commit not to take additional members. Therefore, there is an excessive number of links also in the party equilibrium.

Fig. 2 illustrates the network structure in the party equilibrium. Notice that the structure does not depend on the expected rent, $\psi$.

According to (6), a party equilibrium exists if and only if the number of citizens per politician is not too small or too large, the networking cost and the number of politicians per party boss is sufficiently small and the expected rent from the nomination is sufficiently large. Using the network structure specified in proposition 2, we can establish the payments made by the party bosses to the politicians and the membership fees collected from the citizens\(^{13}\).

5 Welfare

In this section, we compare welfare in the two equilibria - one without the involvement of the parties and the other when the parties are present. We use the aggregate surplus - the sum of the payoffs of those involved - as our welfare measure. The sum of payoffs in the party equilibrium is

$$W^P = n_A\pi^P_A + n_B\pi^P_B + n_C\pi^P_C.$$\(^{13}\)

\(^{13}\)Reported explicitly in the appendix.
Figure 2: Connections in the party equilibrium, $n_c = \gamma = \omega = 2$. 
Without parties the corresponding expression is given by equation (5). These incorporate also the total costs of networking which equal

\[ n_C \omega \left\{ q_A^C \frac{1}{2} (m_{AB}^N(\psi, c, \gamma) + 1)^2 + (1 - q_A)^C \frac{1}{2} (m_{AB}^N(\psi, c, \gamma))^2 \right\} \]

\[ + \gamma \left\{ q_B^C \frac{1}{2} (m_{BA}^N(\psi, c, \gamma) + 1)^2 + (1 - q_B)^C \frac{1}{2} (m_{BA}^N(\psi, c, \gamma))^2 \right\} \]

and

\[ n_C^C \omega \left[ \omega + (\gamma + 1)^2 + \gamma \right] \]

in the no-party equilibrium and in the party equilibrium, respectively. Essentially, all that matters for the welfare comparison are the relative costs of networking in the two types of equilibria.

Notice that, without parties, the expected rent, \( \psi \), appears also on the cost side where it enters through the number of connections\(^{14} m_{BA}^N(\psi, c, \gamma) \). However, in the party equilibrium, the expected rent does not appear on the cost side. Our main finding is the following:

**Theorem 1**

- In regimes (i) and (ii), the gate-keeping role is detrimental for welfare if and only if the stable network without parties is characterized by

\[ m_{BA}^N(\psi, c, \gamma) \leq \sqrt{\frac{(\gamma + 1)^2 + \gamma + \omega}{\gamma(\gamma + 1)}} - 1. \]

- In regimes (iii) and (iv), the gate-keeping role of the parties is detrimental for welfare if the stable network without parties is characterized by

\[ m_{BA}^N(\psi, c, \gamma) \leq \sqrt{\frac{(\gamma + 1)^2 + \gamma + \omega}{\gamma(\gamma + 1)}} - 1, \]

and it is beneficial for welfare if

\[ m_{BA}^N(\psi, c, \gamma) \geq \sqrt{\frac{(\gamma + 1)^2 + \gamma + \omega}{\gamma(\gamma + 1)}}. \]

If \( \sqrt{\frac{(\gamma + 1)^2 + \gamma + \omega}{\gamma(\gamma + 1)}} - 1 < m_{BA}^N(\psi, c, \gamma) < \sqrt{\frac{(\gamma + 1)^2 + \gamma + \omega}{\gamma(\gamma + 1)}} \) then there are threshold values \( \hat{m}_{AB}^{\text{iii}}(\psi, c, \gamma) \) and \( \hat{m}_{AB}^{\text{iv}}(\psi, c, \gamma) \) such that the gate-keeping role of the parties is beneficial for welfare if and only if \( m_{AB}^N(\psi, c, \gamma) \geq \hat{m}_{AB}^{\text{iii}}(\psi, c, \gamma) \) and \( m_{AB}^N(\psi, c, \gamma) \geq \hat{m}_{AB}^{\text{iv}}(\psi, c, \gamma) \), in regimes (iii) and (iv), respectively.

\(^{14}\)The sum of payoffs in the stable network without parties, \( W^N(.) \), is explicitly given in Lemma 2 in the appendix.
• There are parameter values for which the parties are beneficial and parameter values for which the reverse holds.

Theorem 1 is illustrated in Fig. 3 where $\gamma = 2$, $\omega = 2$ and $c = 1$ and the sum of payoffs for the party and its members is given as a function of the expected rent $\psi$.

Given that there are $\omega$ politicians for each party, $\omega \psi$ is the aggregate expected value of nominations within a party. In Fig. 3, $\omega \psi$ is the leftmost straight line of slope equal to two (since Fig. 3 assumes two politicians per party). Some of this value is lost in networking. Thus, the sums of payoffs with and without parties, respectively $W^P$ and $W^N$, lie below $\omega \psi$, the difference indicating the total networking costs. The key finding is that the equilibrium networking costs are unaffected by $\psi$ in the party equilibrium. In principle, if the party was able to capitalize the higher rents as higher rewards from the citizens, the party would be willing to network with a higher number of politicians since marginal value a politician would be higher in that case. Yet competition between parties holds the parties’ marginal rents at bay and prevents the parties from reaping the additional benefits as $\psi$ grows. Thus the networking costs are unaffected by $\psi$ in the party equilibrium. Thus, the distance between $\omega \psi$ and $W^P$ is constant. However, without parties, the costs of networking are increasing in $\psi$ and, therefore, the distance between $\omega \psi$ and $W^N$ increases in $\psi$. While for a small $\psi$, the number of connections is small and without parties the costs of networking are smaller, eventually as $\psi$ increases the networking costs without parties will exceed those in the presence of parties, and the latter will generate a higher welfare than the former.

Fig. 3 also illustrates that the two welfare graphs cross at a unique point, defined as $\hat{\psi}$. When $\psi > \hat{\psi}$, then the party equilibrium is better. This is because higher rents lead to a higher level of networking in the no-party equilibrium. In the party equilibrium, however, the party bosses control the network, competition between parties prevents the parties from reaping the additional benefits due to higher $\psi$, and the cost of networking is constant w.r.t. $\psi$.

The flat parts in the graph $W^N$ correspond to regime (iii). So the case in the figure corresponds to $\sqrt{\frac{(\gamma + 1)^2 + \gamma + \omega}{\gamma (\gamma + 1)}} > m^N_{BA}(\psi, c) > \sqrt{\frac{(\gamma + 1)^2 + \gamma + \omega}{\gamma (\gamma + 1)}} - 1$ but since the difference $W^N - W^P$ is monotone in the link numbers even in regimes (iii) and (iv), the crossing point is uniquely defined and we can characterize the welfare properties as is implicitly done in Theorem 1.

Proposition 1 and Theorem 1 allow us to show how the welfare effect of the parties, $W^P - W^N$, depends on the expected rent, $\psi$, and on the networking costs, $c$.

**Proposition 3** An increase (a decrease) in the expected rent $\psi$ or a decrease (an increase) in the network cost parameter $c$ weakly increases (decreases) the benefits generated by the gate-keeping role of the parties.

**Proof.** Suppose that a fraction $q_A$ of the politicians have $m_{AB} + 1$ links and a fraction $1 - q_A$ of the politicians have $m_{AB}$ links. Similarly assume that a fraction $q_B$
Figure 3: Comparison of welfare associated with 2 politicians and 4 citizens with and without parties.

of the politicians have $m_{BA} + 1$ links and a fraction $1 - q_B$ of the politicians have $m_{BA}$ links. Then the total networking costs in the no-party equilibrium are

$$n_C \omega \{q_A \frac{c}{2}(m_{AB} + 1)^2 + (1 - q_A) \frac{c}{2}(m_{AB})^2 + \gamma[q_B \frac{c}{2}(m_{BA} + 1)^2 + (1 - q_B) \frac{c}{2}(m_{BA})^2]\}$$

and those in the party equilibrium are $n_C \frac{c}{2} \omega[\omega + (\gamma + 1)^2 + \gamma]$.

Thus, the no-party equilibrium yields a higher welfare if and only if

$$q_A(2m_{AB} + 1) + (m_{AB})^2 + \gamma[q_B(2m_{BA} + 1) + (m_{BA})^2]$$

$$< [\omega + (\gamma + 1)^2 + \gamma],$$

and comparative welfare gains only depend on the number of links in the no-party equilibrium, $m_{BA}^N(\psi, c, \gamma)$ and $m_{AB}^N(\psi, c, \gamma)$, which are increasing in $\psi$ and decreasing in $c$. Thus $\partial(W^N - W^P)/\partial \psi < 0$ and $\partial(W^N - W^P)/\partial c > 0.$

We find that the involvement of the parties in the nominations is bound to result in a relatively higher social welfare, if the value of the rent increases and if the cost of networking decreases. The number of links is increasing in $\psi$ in the no-party equilibrium and thus the welfare cost of the equilibrium without parties is increasing but the costs are unchanged in the party equilibrium where the network structure does not respond to the level of rents. So a higher $\psi$ implies lower $W^N - W^P$. When $c$ increases, the number of links decreases in the no party equilibrium. The costs in the party equilibrium
increase at a rate proportional to the constant number of links in the party network. In the no-party equilibrium, the fact that the number of links is reduced at the margin due to a higher $c$ brings about an effect that counteracts the cost-increasing effect due to higher $c$ for inframarginal links proportional to the number of links. Thus a higher $c$ tends to favor the no-party equilibrium.

6 Conclusion

In this paper, we analyze the welfare implications of political parties taking a role in the distribution of nominations for non-ideological jobs and positions of trusts. We take as our starting point that there are anti-corruption laws which prevent political nominations from being sold. These laws still allow rent-seeking citizens to invest in good connections to nominating politicians. They spend time with the politicians by taking part in fund-raising events, volunteering their time in campaigns and by offering lunches and entertainment. Competition for the politicians’ attention results in wasteful networking. As the party can require its politicians to only nominate from a subgroup of the party members, the political parties can improve efficiency by restricting the incentives to rent-seeking through connections.

It should be highlighted that the anti-corruption laws also restrict the activities of political parties. They are not allowed to sell the nominations either, but only to receive membership payments and allocate funds to politicians’ campaigns. Even political parties are unable to fully eliminate wasteful networking, as they cannot commit to restricting the number of members.\footnote{Allowing political parties to pre-commit to not admit additional members would disenfranchise those citizens not belonging to the selected few from fully participating in the political life.} Social desirability of the involvement of the political parties in the non-ideological nominations increases as the cost of networking decreases and as the expected value of the nominations increases. This suggests a novel welfare motivation for the role of political parties in such nominations.

Our framework raises several topics for further research. First, we could endogenize the identity of the party bosses by presenting an overlapping-generations framework in which the party bosses arise from senior politicians.

Second, we could allow for citizens to differ in their skills and preferences. In that case, politicians have an incentive to be connected with the citizens both in order to search for a competent one and to cash in the citizens’ desire for nomination. Our main insight should remain: politicians would like to sell more connections than would be optimal from the efficiency perspective. Political parties could still improve efficiency by limiting the extent of connections when rents are sufficiently high. There could also be a difference in how political parties distribute rents: Elinder and Jordahl (2013) find that center-right majorities engage in more outsourcing than left majorities in Sweden.

Third and finally, we abstract from the role of ideological considerations.
politicians and citizens might join the parties purely for ideological reasons. In a richer model, the citizens would differ in their ideology, and the politicians and the political parties would differ in their candidate valuations. In that case, the political parties could face a choice between the ideologically more appealing candidates and those willing to pay more for gaining access. Such trade-offs and heterogeneity in the ideological importance of positions could help to explain why some positions are typically filled by ideological party members, while others are used as rewards for contributors. If both potential party members and citizens would differ in their ideology, citizens could organize around politicians also based on ideological proximity.

7 Appendix

7.1 Proof of proposition 2

Proof. 1) Let us first assume that each party has $\gamma$ citizens for each politician connected to it, that is $\mu_{CB} = \gamma m_{CA}$. The equilibrium reward must be such that the party is indifferent whether to have an additional connection or not. An additional connection to a citizen would increase the networking costs of the politician to whom the citizen would be connected from $\frac{c}{2}(\gamma + 1)^2$ to $\frac{c}{2}(\gamma + 2)^2$. In addition, the party would have to pay $\frac{c}{2}$ for the new citizen’s networking cost as the party bears all networking costs. The marginal increase in the networking costs then equals $\frac{c}{2}(2\gamma + 4) = c(\gamma + 2)$. For any party, the net gain from supplying a connection to one more citizen cannot be positive in equilibrium. Hence, $r_{BC}^P \leq c(\gamma + 2)$. On the other hand, it is not possible that the net gain is negative either, $r_{BC}^P < c(\gamma + 2)$, since then each party could increase the reward that a citizen has to pay up to $c(\gamma + 2)$. The citizen remains with his party even with the higher reward since, for every party $r_{BC}^P \leq c(\gamma + 2)$, and hence no party strictly prefers offering a connection to an additional citizen. Thus,

$$r_{BC}^P = c(\gamma + 2).$$

(7)

2) Given that each party has $m_{CA}$ politicians, the equilibrium number of citizens per party is $m_{CA}\gamma$. To see this, suppose to the contrary that there are two political parties, $C'$ and $C''$ such that the number of citizens connected to the two political parties are such that $\frac{\mu_{CB}'}{m_{CA}} < \frac{\mu_{CB}''}{m_{CA}}$. Then, since all citizens and politicians are connected and $n_B = \gamma n_A = \gamma n_C$, we can choose two political parties so that $\frac{\mu_{CB}'}{m_{CA}} < \gamma < \frac{\mu_{CB}''}{m_{CA}}$. However, then using the cheapest structure described in point (1) of the proof, every politician connected to $C''$ must have $\gamma + 1$ connections or less. Denote the largest number of connections of a politician in party $C''$ by $m''$. For the party $C'$, on the other hand, there must be a politician for whom the number of connections $m'$ is strictly
greater than $\gamma + 1$. Hence,

$$\frac{c}{2}(m'' + 1) \leq \frac{c}{2}(\gamma + 1) < \frac{c}{2}(m' + 1).$$

(8)

The reward $r'_{BC}$ of the party $C'$ must be higher than or equal to $c(m'+2)$. Otherwise, the last additional connection does not provide positive profit. However, for $C''$ the marginal cost is lower and therefore,

$$r'_{BC} \geq c(m' + 2) > c(m'' + 2).$$

Thus, party $C''$ makes a profit by providing a cheaper additional connection to a customer of $C'$ and the customer has a higher or equal probability of receiving the nomination with $C''$ than with $C'$ and this cannot be an equilibrium. This is a contradiction. Hence, $\frac{\omega_{CB}}{m'_{CA}} = \frac{\omega_{CB}}{m''_{CA}} = \gamma$.

3) Let us now show that the equilibrium reward $r^P_{CA}$ satisfies

$$r^P_{CA} = \frac{c(\gamma^2 + \gamma - 1)}{2} - \frac{c}{2}(2m^P_{CA} + 1).$$

(9)

The benefits to the party are the payments from all citizens connected to the politicians, $m^P_{CA}\gamma r^P_{BC}$.

The costs include the payment made to the politicians $m^P_{CA}r^P_{CA}$, the networking costs of politicians paid by political parties, $m^P_{CA}\frac{\xi}{2}(\gamma + 1)^2$, the networking costs of the citizens connected to the politicians of the party, $m^P_{CA}\frac{\xi}{2}$, and the party’s own networking costs to the politicians $\frac{\xi}{2}(m^P_{CA})^2$. In equilibrium, the marginal benefit from connections to politicians must equal its marginal cost, that is

$$\gamma r^P_{BC} = \frac{c}{2}\gamma + r^P_{CA} + \frac{c}{2}(\gamma + 1)^2 + \frac{c(2m^P_{CA} + 1)}{2}.$$

(10)

Substituting from (7), the payment $r^P_{CA}$ is given by (9).

4) Let us now show that a network structure, where some party’s amount of connections, $m_{CA}$, differs from $\omega$, cannot be part of an equilibrium. If there is such a party then, since all politicians are connected to a party, there must be parties $C''$ and $C'$ with $m'_{CA} < \omega < m''_{CA}$, and thus $m''_{CA} \geq m'_{CA} + 2$. The party $C''$ is not willing to pay more than $r''_{CA} = \frac{c(\gamma^2 + \gamma - 1)}{2} - \frac{c}{2}(2m''_{CA} + 1)$ to the politicians connected to it. Otherwise, the last additional politician would deteriorate the payoff of $C''$. However, $C'$ can recruit a politician connected to $C''$ with a positive profit, since

$$r'_{CA} \leq \frac{c(\gamma^2 + \gamma - 1)}{2} - \frac{c}{2}(2m''_{CA} + 1) < \frac{c(\gamma^2 + \gamma - 1)}{2} - \frac{c}{2}(2m'_{CA} + 1)$$

and $C'$ can afford paying $r'_{CA} + \varepsilon$ for a sufficiently small $\varepsilon > 0$. Hence, $m_{CA} \neq \omega$ cannot be an equilibrium.
The equilibrium reward $r_{BC}^P$ is given by equation (7) in part 1) of the proof. The equilibrium reward,

$$r_{CA}^P = \frac{c}{2}(\gamma^2 + \gamma - 2\omega - 2),$$

(11)

now follows from substituting $r_{BC}^P$ and $m_{CA}^P = \omega$ into (10) and rearranging. Thus, $\pi_A^P \geq 0$ and $\pi_B^P \geq 0$ if and only if

$$\frac{\psi}{c} \geq \gamma(\gamma + 2) \geq 2\omega + 2 + \gamma.$$  (12)

7.2 Proof of theorem 1

In the proof of the theorem, we need the following two lemmas which are proved in Miettinen and Poutvaara (2014) and thus stated here without a proof. The definition of pairwise stability with transfers that we use is provided in Miettinen and Poutvaara (2014).

7.2.1 Lemma 1

There are different regimes of stable networks. Lemma 1 lists the parameter values for which each regime prevails given numbers of connections in the stable network.

**Lemma 1** Given $c$ and $\gamma$ and that $\psi \geq c\gamma^2$, one and only one of the regimes prevails for each $\psi$.

- **Regime (i)** where citizens are connected with $m_{BA}^N$ politicians and politicians are connected with $\gamma m_{BA}^N$ citizens prevails if and only if

  $$c(m_{BA}^N)^2\gamma(\gamma + 1) - cm_{BA}^N \gamma \leq \psi \leq c\gamma(\gamma + 1)(m_{BA}^N)^2.$$  

  If $\psi$ is increased above the upper bound, one enters an interval belonging to regime (ii) with each citizen having $m_{BA}^N$ connections.

- **Regime (ii)** where citizens are connected with $m_{BA}^N$ politicians and politicians are connected with $\gamma m_{BA}^N$ citizens prevails if and only if

  $$c\gamma(\gamma + 1)(m_{BA}^N)^2 < \psi \leq c\gamma m_{BA}^N(m_{BA}^N(\gamma + 1) + 1).$$  

  If $\psi$ is increased above the upper bound, one enters an interval belonging to regime (iii) with each citizen having $m_{BA}^N$ or $m_{BA}^N + 1$ connections.
• Regime (iii) where citizens have $m_{BA}^N$ or $m_{BA}^N + 1$ connections whereas politicians have $m_{AB}^N$ or $m_{AB}^N + 1$ connections where $\gamma m_{BA}^N \leq m_{AB}^N < \gamma (m_{BA}^N + 1)$ prevails if and only if

$$cm_{AB}^N (m_{AB}^N + m_{BA}^N + 1) < \psi < c(m_{AB}^N + 1)(m_{AB}^N + m_{BA}^N + 1).$$

If $\psi$ is increased above the upper bound and

- if $m_{AB}^N < \gamma (m_{BA}^N + 1) - 1$,
  one enters an interval belonging to regime (iv) with $m_{AB}^N + 1$ connections for politicians.
- if $m_{AB}^N = \gamma (m_{BA}^N + 1) - 1$,
  one enters an interval belonging to regime (i) with $m_{BA}^N + 1$ connections for citizens.

• Regime (iv) where citizens have $m_{BA}^N$ or $m_{BA}^N + 1$ connections whereas politicians have $m_{AB}^N$ connections where $\gamma m_{BA}^N + 1 \leq m_{AB}^N < \gamma (m_{BA}^N + 1)$ prevails if and only if

$$cm_{AB}^N (m_{AB}^N + m_{BA}^N) \leq \psi \leq cm_{AB}^N (m_{AB}^N + m_{BA}^N + 1).$$

If $\psi$ is increased above the upper bound, one enters an interval belonging to regime (iii) with politicians mixing between $m_{AB}^N$ and $m_{AB}^N + 1$ connections.

**Proof.** See Miettinen and Poutvaara (2014)

7.2.2 Lemma 2

**Lemma 2** Payoffs and the sum of payoffs in the stable network are non-negative and given by

\[
\begin{align*}
\pi_A^N &= \psi - \frac{c}{2}\gamma m_{BA}^N (\gamma m_{BA}^N + 2m_{BA}^N - 1) \\
\pi_B^N &= \frac{c}{2} m_{BA}^N (m_{BA}^N - 1) \\
W^N &= \left[\psi - \frac{c}{2}\gamma (m_{BA}^N)^2 (\gamma + 1)\right] n_A
\end{align*}
\]

in regime (i);

\[
\begin{align*}
\pi_A^N &= \frac{c}{2}\gamma m_{BA}^N (\gamma m_{BA}^N + 1) \\
\pi_B^N &= \psi - \frac{c}{\gamma} m_{BA}^N (2\gamma m_{BA}^N + 1 + m_{BA}^N) \\
W^N &= \left[\psi - \frac{c}{2}\gamma m_{BA}^N (\gamma m_{BA}^N + m_{BA}^N)\right] n_A
\end{align*}
\]
in regime (ii);
\[ \pi^N_A = \frac{c}{2} m_{AB}^N (m_{AB}^N + 1) \]
\[ \pi^N_B = \frac{c}{2} m_{BA}^N (m_{BA}^N + 1) \]
\[ W^N = \left[ \frac{c}{2} m_{AB}^N (m_{AB}^N + 1) + \frac{c}{2} \gamma m_{BA}^N (m_{BA}^N + 1) \right] \]

in regime (iii); and
\[ \pi^N_A = \psi - \frac{c}{2} m_{AB}^N (2m_{BA}^N + m_{AB}^N + 1) \]
\[ \pi^N_B = \frac{c}{2} m_{BA}^N (m_{BA}^N + 1) \]
\[ W^N = \left[ \psi + \frac{c}{2} \gamma m_{BA}^N (m_{BA}^N + 1) - \frac{c}{2} m_{AB}^N (2m_{BA}^N + m_{AB}^N + 1) \right] \]

in regime (iv).

**Proof.** See Miettinen and Poutvaara (2014) ■

### 7.2.3 Proof of the theorem

**Proof.**

The welfare in the no party equilibrium in regimes (i), (ii), (iv) equals
\[ W^N = n_C \omega \{ \psi - \frac{c}{2} [(m_{AB}^N)^2 + \gamma (m_{BA}^N)^2] \}. \]
(See also Lemma 2 in Miettinen & Poutvaara, 2014).

The welfare in the party equilibrium equals
\[ W^P = n_C \omega \{ \psi - \frac{c}{2} [\omega + (\gamma + 1)^2 + \gamma] \}. \]

The former is higher if and only if
\[ (m_{AB}^N)^2 + \gamma (m_{BA}^N)^2 < \omega + (\gamma + 1)^2 + \gamma \] (13)

which simply states that the networking costs are lower in the no-party equilibrium than in the party equilibrium.

In regimes (i) and (ii) all politicians have the same number of connections and all the citizens have the same number of connections and by market clearing \( m_{AB}^N = \gamma m_{BA}^N \). Thus in those regimes, the no-party equilibrium generates a higher welfare, i.e. \( W^N - W^P \geq 0 \), if and only if
\[ m_{BA}^N \leq \sqrt{\frac{(\gamma + 1)^2 + \gamma + \omega}{\gamma(\gamma + 1)}}. \] (14)
If the parameter values are such that regime (i) or (ii) prevails, then the claim in the first bullet point of the theorem follows directly from (14). If the prevailing regime is (iii) or (iv), we proceed as follows: we hypothetically increase or decrease $\psi$ as little as possible but so that a nearby regime (i) or regime (ii) interval is reached, respectively. This can be done by the alternation rule of regimes pointed out in Lemma 1. We check whether (14) holds at that hypothetical situation and since both $m_{BA}^N(\psi, c, \gamma)$ and $W^N(\psi, c, \gamma, \omega) - W^P(\psi, c, \gamma, \omega)$ are monotone in $\psi$, we are able to make a claim about $W^N(\psi, c, \gamma, \omega) - W^P(\psi, c, \gamma, \omega)$ with the original $\psi$.

Remember that if regime (iii) or (iv) with $m_{BA}^N(\psi, c, \gamma)$ prevails, then some citizens have $m_{BA}^N(\psi, c, \gamma)$ connections whereas some others have $m_{BA}^N(\psi, c, \gamma) + 1$ connections. On the other hand, if regime (i) or (ii) with $m_{BA}^N(\psi, c, \gamma)$ prevails, then all citizens have $m_{BA}^N(\psi, c, \gamma)$ connections.

Assume regime (iii) or (iv) prevails and denote the prevailing expected rent by $\bar{\psi}$. Consider $\psi' < \bar{\psi}$ where $\psi'$ is the largest $\psi$ such that regime (ii) with $m_{BA}^N(\tilde{\psi}, c, \gamma)$ connections prevails (by the alternation rule of the regimes in lemma 1, regime (ii) with $m_{BA}^N(\tilde{\psi}, c, \gamma)$ prevails for an interval of values of $\psi$ smaller than $\bar{\psi}$). Thus, $m_{BA}^N(\tilde{\psi}, c, \gamma) = m_{BA}^N(\psi', c, \gamma)$.

Now if $m_{BA}^N(\psi', c, \gamma) > \sqrt{\frac{(\gamma+1)^2 + \gamma + \omega}{\gamma(\gamma+1)}}$ then $W^N(\psi', c, \gamma, \omega) - W^P(\psi', c, \gamma, \omega) < 0$ and since $W^N(\psi, c, \gamma, \omega) - W^P(\psi, c, \gamma, \omega)$ is decreasing in $\psi$ and $\psi' < \tilde{\psi}$, $W^N(\tilde{\psi}, c, \gamma, \omega) - W^P(\tilde{\psi}, c, \gamma, \omega) < 0$.

Consider now $\psi' > \tilde{\psi}$ where $\psi'$ is the smallest $\psi$ such that regime (i) prevails with $m_{BA}^N(\tilde{\psi}, c, \gamma) + 1$ connections (by the alternation rule of the regimes in lemma 1, regime (i) with $m_{BA}^N(\tilde{\psi}, c, \gamma) + 1$ prevails for an interval of values of $\psi$ greater than $\bar{\psi}$). Thus $m_{BA}^N(\psi', c, \gamma) = m_{BA}^N(\psi', c, \gamma) + 1$. Now if

$$m_{BA}^N(\psi', c, \gamma) \leq \sqrt{\frac{(\gamma+1)^2 + \gamma + \omega}{\gamma(\gamma+1)}},$$

then $W^N(\psi', c, \gamma, \omega) - W^P(\psi', c, \gamma, \omega) \geq 0$ and since $W^N(\psi, c, \gamma, \omega) - W^P(\psi, c, \gamma, \omega)$ is decreasing in $\psi$ and $\psi' > \bar{\psi}$, $W^N(\tilde{\psi}, c, \gamma, \omega) - W^P(\tilde{\psi}, c, \gamma, \omega) \geq 0$. However, (15) is equivalent to

$$m_{BA}^N(\tilde{\psi}, c, \gamma) \leq \sqrt{\frac{(\gamma+1)^2 + \gamma + \omega}{\gamma(\gamma+1)} - 1},$$

where $\bar{\psi}$ is the original value (this is because $m_{BA}^N(\psi', c, \gamma) + 1 = m_{BA}^N(\psi', c, \gamma)$).

We know that networking costs and the amounts of links are constant in the party equilibrium and increasing in the no-party equilibrium. Thus in regime (i) or (ii), if $m_{BA}^N(\psi', c, \gamma) > \sqrt{\frac{(\gamma+1)^2 + \gamma}{\gamma(\gamma+1)}}$ then for all $\psi \geq \psi'$ the party equilibrium is better for welfare, independently of the regime. And if $m_{BA}^N(\psi', c, \gamma) < \sqrt{\frac{\omega+(\gamma+1)^2 + \gamma}{\gamma(\gamma+1)}}$ then for all
the no-party equilibrium is better for welfare, independently of the regime. If the regime is (iii) or (iv) and if 
\[ \sqrt{\frac{(\gamma+1)^2+\gamma\omega}{\gamma(\gamma+1)}} > m_{BA}^{N}(\tilde{\psi}, c, \gamma) \sqrt{\frac{(\gamma+1)^2+\gamma\omega}{\gamma(\gamma+1)}} - 1, \]
we do not quite know whether \( W^{N}(\tilde{\psi}, c, \gamma, \omega) - W^{P}(\tilde{\psi}, c, \gamma, \omega) \) is positive or negative and we need further conditions.

Let’s use the following
\[ \sum_{m_{AB}} m_{AB}q_{m_{AB}} = \gamma \sum_{m_{BA}} m_{BA}q_{m_{BA}} \]
and compare the total networking costs in the no-party equilibrium
\[ \left[ \sum_{m_{AB}} \frac{c}{2}(m_{AB})^2q_{m_{AB}} + \gamma \sum_{m_{BA}} \frac{c}{2}(m_{BA})^2q_{m_{BA}} \right] \]
with those in the party equilibrium
\[ \omega_{nC}\frac{c}{2}[\omega + (\gamma + 1)^2 + \gamma]. \]
The type of equilibrium with lower networking costs has a higher welfare.

In regimes (i) and (ii) \( m_{AB} = \gamma m_{BA} \) and thus \( \sum_{m_{AB}} m_{AB}q_{m_{AB}} = \gamma \sum_{m_{BA}} m_{BA}q_{m_{BA}} \) amounts to the total networking costs
\[ \omega_{nC}\frac{c}{2}(\gamma m_{BA})^2 + \gamma \frac{c}{2}(m_{BA})^2]. \]
Thus the no-party equilibrium has a higher welfare if and only if
\[ \omega_{nC}\frac{c}{2}(\gamma m_{BA})^2 + \gamma \frac{c}{2}(m_{BA})^2 < \omega_{nC}\frac{c}{2}[\omega + (\gamma + 1)^2 + \gamma] \]
or equivalently
\[ m_{BA} < \sqrt{\frac{\omega + (\gamma + 1)^2 + \gamma}{\gamma(\gamma + 1)}} \]
which is precisely the condition in Theorem 1.

In regime (iv) \( \sum_{m_{AB}} m_{AB}q_{m_{AB}} = \gamma \sum_{m_{BA}} m_{BA}q_{m_{BA}} \) amounts to
\[ m_{BA}^{N} = \gamma[q_{B}(m_{BA}^{N} + 1) + (1 - q_{B})m_{BA}^{N}] \] (17)
resulting in total networking costs of
\[ \omega_{nC}\frac{c}{2}(\gamma(q_{B}(m_{BA}^{N} + 1) + (1 - q_{B})m_{BA}^{N}))^2 + \gamma[q_{B}\frac{c}{2}(m_{BA}^{N} + 1)^2 + (1 - q_{B})\frac{c}{2}(m_{BA}^{N})^2]. \]
Thus the no-party equilibrium has a higher welfare if and only if
\[ \omega_{nC}\frac{c}{2}(\gamma(q_{B}(m_{BA}^{N} + 1) + (1 - q_{B})m_{BA}^{N}))^2 + \gamma[q_{B}\frac{c}{2}(m_{BA}^{N} + 1)^2 + (1 - q_{B})\frac{c}{2}(m_{BA}^{N})^2] \]
We can solve $q_B$ from equation (17) yielding

$$q_B = \frac{m_{AB}^N}{\gamma} - m_{BA}^N.$$  

Substituting this into the inequality yields

$$(m_{AB}^N)^2 + m_{AB}^N m_{BA}^N + (m_{AB}^N - \gamma m_{BA}^N)(m_{BA}^N + 1)$$

$$< [\omega + (\gamma + 1)^2 + \gamma],$$

where $\gamma m_{BA}^N < m_{AB}^N < \gamma(m_{BA}^N + 1)$ by the equilibrium regime condition. For all but

$$\sqrt{\frac{(\gamma + 1)^2 + \gamma + \omega}{\gamma(\gamma + 1)}} > m_{BA}^N > \sqrt{\frac{(\gamma + 1)^2 + \gamma + \omega}{\gamma(\gamma + 1)}} - 1$$

we know whether the no-party or the party equilibrium is better. The left hand side of the inequality is increasing in $m_{AB}^N$. Thus in regime (iv), the no-party equilibrium yields a higher welfare if and only if

$$\sqrt{\frac{(\gamma + 1)^2 + \gamma + \omega}{\gamma(\gamma + 1)}} > m_{BA}^N > \sqrt{\frac{(\gamma + 1)^2 + \gamma + \omega}{\gamma(\gamma + 1)}} - 1$$

where $m_{BA}^N < m_{AB}^N < \gamma(m_{BA}^N + 1)$ and

$$(m_{AB}^N)^2 + m_{AB}^N m_{BA}^N + (m_{AB}^N - \gamma m_{BA}^N)(m_{BA}^N + 1)$$

$$< [\omega + (\gamma + 1)^2 + \gamma],$$

or if

$$\sqrt{\frac{(\gamma + 1)^2 + \gamma + \omega}{\gamma(\gamma + 1)}} > m_{BA}^N > \sqrt{\frac{(\gamma + 1)^2 + \gamma + \omega}{\gamma(\gamma + 1)}} - 1$$

In regime (iii) $\sum_{m_{AB}} m_{AB} q_{m_{AB}} = \gamma \sum_{m_{BA}} m_{BA} q_{m_{BA}}$ amounts to

$$q_A(m_{AB}^N + 1) + (1 - q_A)(m_{AB}^N) = \gamma[q_B(m_{BA}^N + 1) + (1 - q_B)m_{BA}^N]$$

or equivalently

$$q_B = \frac{m_{AB}}{\gamma} + \frac{q_A}{\gamma} - m_{BA},$$

so that the total networking costs in the no-party equilibrium are

$$n_{C}\omega \{q_A \frac{C}{2} (m_{AB}^N + 1)^2 + (1 - q_A) \frac{C}{2} (m_{AB}^N)^2 + \gamma [q_B \frac{C}{2} (m_{BA}^N + 1)^2 + (1 - q_B) \frac{C}{2} (m_{BA}^N)^2] \}$$

or equivalently

$$n_{C}\omega \{q_A \frac{C}{2} (2m_{AB}^N + 1) + \frac{C}{2} (m_{AB}^N)^2 + \gamma [q_B \frac{C}{2} (2m_{BA}^N + 1) + \frac{C}{2} (m_{BA}^N)^2] \}.$$  

Thus the no-party equilibrium yields a higher welfare if and only if

$$q_A(2m_{AB}^N + 1) + (m_{AB}^N)^2 + \gamma[q_B(2m_{BA}^N + 1) + (m_{BA}^N)^2]$$

$$< [\omega + (\gamma + 1)^2 + \gamma].$$
Substituting (19)

\[ q_A(2m_{AB}^N + 1 + (m_{AB})^2 + (m_{AB} + q_A - \gamma m_{BA})(2m_{BA}^N + 1) + \gamma (m_{BA}^N)^2 \]

\[ < [\omega + (\gamma + 1)^2 + \gamma], \]

or equivalently

\[ q_A(2m_{AB}^N + 1 + 2m_{BA}^N + 1) + (m_{AB}^N)(m_{AB}^N + m_{BA}^N) + (m_{AB}^N - \gamma m_{BA})(m_{BA}^N + 1) \]

\[ < [\omega + (\gamma + 1)^2 + \gamma]. \]

The equilibrium indifference condition for rent-seekers gives

\[ q_A = (m_{AB}^N + 1)[1 - \frac{\xi}{2}(2m_{BA}^N + 1) + \frac{\xi}{2}(2m_{AB}^N + 1)]m_{AB}^N \]

or equivalently

\[ q_A = (m_{AB}^N + 1)[1 - \frac{\xi}{2}(2m_{BA}^N + 1 + 2m_{AB}^N + 1) - \frac{cm_{AB}}{\psi}m_{AB}^N] \]

(20)

(This results after simple algebraic derivations; Proof of Lemma 1 in Miettinen & Poutvaara, 2014). The derivative of this expression with respect to \( m_{AB}^N \) equals 

\[ 1 - \frac{\xi}{2}(2m_{BA}^N + 1 + 2m_{AB}^N + 1) - \frac{cm_{AB}}{\psi}m_{AB}^N \]

which is positive if \( \psi > \frac{\xi}{2}(2m_{BA}^N + 1 + 2m_{AB}^N + 1) + (m_{AB}^N + 1)cm_{AB}^N \). But the regime boundary condition states that \( \psi > \frac{m_{AB}^N c}{2}(2m_{BA}^N + 1 + 2m_{AB}^N + 1) + (m_{AB}^N + 1)cm_{AB}^N \) where the latter inequality is equivalent with \( m_{AB}^N > \frac{(m_{AB}^N + 1) + (2m_{BA}^N + 1)}{2m_{BA}^N} \). The right-hand side is at most 

\[ \frac{\gamma (m_{BA}^N + 1)}{2m_{BA}^N} + 1 + \frac{1}{m_{BA}^N} \] and the left-hand side is at least \( \gamma m_{BA}^N \) resulting in inequality 

\[ m_{BA}^N > \frac{(m_{AB}^N + 1)}{2m_{BA}^N} + \frac{1}{\gamma} \] which holds for \( m_{BA}^N \geq 2 \) when \( \gamma \geq 2 \). Thus the derivative of the left-hand side of the expression

\[ q_A(2m_{AB}^N + 1 + 2m_{BA}^N + 1) + (m_{AB}^N)(m_{AB}^N + m_{BA}^N) + (m_{AB}^N - \gamma m_{BA})(m_{BA}^N + 1) \]

\[ < [\omega + (\gamma + 1)^2 + \gamma]. \]

is increasing in \( m_{AB}^N \) and thus the inequality implicitly defines an upper bound for \( m_{AB}^N \) such that in regime (iii) the no-party equilibrium is preferred if and only if \( m_{AB}^N \) is below the upper bound. This completes the proof of the theorem. ■

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References


